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Asymptotic Normality of L_1 -Estimators in a Partly Linear Model

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Abstract: Consider the partly linear model $Y = X'\beta_0 + g(T) + e$, where β_0 is a $k \times 1$ vector of unknown parameters, $g(\cdot)$ is an unknown smooth function and e is an unobserved disturbance. A piecewise polynomial $g_n(\cdot)$ is proposed to approximate g and the least absolute deviation estimator of β_0 is obtained. Under milder conditions the asymptotic distribution of the estimator of β_0 is derived.

Key words: partly linear model, least absolute deviation, asymptotic normality

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部分线性模型中 L_1-估计量的渐近正态性

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[摘要] 给定部分线性模型 $Y = X'\beta_0 + g(t) + e$, 其中 β_0 是一 $k \times 1$ 未知参数向量, $g(\cdot)$是一未知的光滑函数, e 为一随机误差. 我们先用一逐段多项式 g_n 逼近未知函数 g, 然后用最小一乘法得到未知参数 β_0 的最小绝对偏差估计量 $\hat{\beta}$. 在较弱的条件下我们 推导了估计量 $\hat{\beta}$ 的渐近正态性.

[关键词] 部分线性模型,最小绝对偏差,渐近正态性

0 Introduction

Consider the following partly linear model given by

$$Y = X'\beta_0 + g(T) + e, \tag{1}$$

where $X' = (x_1, \cdots, x_k)$ and T are explanatory variables, $\boldsymbol{\beta}_0$ is a $k \times 1$ vector of unknown parameters, $g(\cdot)$ is an unknown smooth function of T in $[0,1]$, e is the random error with median 0, (X, T) and e are independent. The partially linear model was first introduced in paper [1] to study the effect of weather on electricity demand and further studied by refs. [2,3], etc. When $EX = 0$, ref. [3] derived the asymptotic normality of the L_1-norm estimators of β_0 using the B-spline method. Let $\{X_i = (X_{i1}, \cdots, X_{ik})', T_i, Y_i, (1 \leqslant i \leqslant n)\}$ denote a sample of size n from (1). In this paper, we use a piecewise polynomial g_n to approximate g, then minimize

$$\sum_{i=1}^{n} | Y_i - X'_i\beta - g_n(T_i) |$$

to obtain L_1-estimators $\hat{\beta}$ of β_0. Under general conditions we show that $\hat{\beta}$ is asymptotic normal. Morever, comparing to paper [3], our method is more simple and easier to understand and carry out in practice.

1 Main Results

First we make the following assumptions.

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A1. The distribution of T is absolutely continuous and its density $w(t)$ satisfies $0 < b \leqslant \inf_{0\leqslant t\leqslant 1} w(t) \leqslant \sup_{0\leqslant t\leqslant 1} w(t) \leqslant B < +\infty$.

A2. Let $0 < \gamma \leqslant 1$ and $0 < M$, g is an m-times continuously differential function such that $|g^{(m)}(t_2) - g^{(m)}(t_1)| \leqslant M|t_2 - t_1|^{\gamma}$, for $0 \leqslant t_1, t_2 \leqslant 1$. Think of $p = m + \gamma$ as a measure of the smoothness of the function g.

A3. The median of the distribution of random error e is 0, the distribution of e has density $f(x)$ in a neighborhood of 0, $f(0) > 0$ and is continuous at $x = 0$.

A4. $EX = 0$, $EXX' = (\sigma_{ij})_{k\times k}$ is a positive definite matrix, and X and T are independent.

A4′. EX exists and $E(X - EX)(X - EX)' = (\bar{\sigma}_{ij})_{k\times k}$ is a positive definite matrix, and X and T are independent.

A5. $\max\limits_{1\leqslant i\leqslant n} \|X_i\| = O_p(n^{(p-1/2)/(2p+1)}/\log n)$.

A6. There exist two positive constants c_1 and c_2 such that, $f(x)$ satisfies $|f(x) - f(0)| \leqslant c_2|x|$ for all $x \in [-c_1, c_1]$.

Given a positive integer M_n, we split equally $[0,1]$ into M_n subintervals. The length of every subinterval is $2h = 1/M_n$. Let $I_{n\nu} = [(\nu-1)/M_n, \nu/M_n)$, $1 \leqslant \nu \leqslant M_n - 1$, $I_{nM_n} = [1 - 1/M_n, 1]$. Let d_ν denote the center of the interval $I_{n\nu}$ and $\boldsymbol{A}(t)$ denote a $(m+1)M_n \times 1$ vector such that for $t \in I_{n\nu}$, $\nu = 1, \cdots, M_n$,

$$\boldsymbol{A}(t_i)' = (0, \cdots, 0, 1, (t_i - d_\nu)/h, \cdots, [(t_i - d_\nu)/h]^m, 0, \cdots, 0),$$

Set $\boldsymbol{\alpha}' = (\alpha_1, \cdots, \alpha_{m+1}, \cdots, \alpha_{(m+1)M_n})$, $\boldsymbol{g}_n(t) = \boldsymbol{A}(t)'\boldsymbol{\alpha}$ and

$$\boldsymbol{\alpha}'_0 = (g(d_1), \cdots, h^m g^{(m)}(d_1)/m!, \cdots, g(d_{M_n}), \cdots, h^m g^{(m)}(d_{M_n})/m!).$$

Let

$$\sum_{i=1}^{n} |Y_i - X'_i\hat{\beta} - A(T_i)'\hat{\alpha}| = \min_{\beta,\alpha} \sum_{i=1}^{n} |Y_i - X'_i\beta - A(T_i)'\alpha|, \tag{2}$$

then we have the following theorems.

Theorem 1 Suppose that A1-A6 hold and that $M_n \sim n^{1/(2p+1)}$ and $p > 3/2$. Then

$$2f(0)(EXX')^{1/2}\sqrt{n}(\hat{\beta} - \beta_0) \to_d N(0, I_k).$$

Theorem 2 Suppose that A1-A3, A4′, A5-A6 hold and that $M_n \sim n^{1/(2p+1)}$ and $p > 3/2$. Then

$$2f(0)(E(X - EX)(X - EX)')^{1/2}\sqrt{n}(\hat{\beta} - \beta_0) \to_d N(0, I_k).$$

2 Proofs of the Theorems

Let L_n be a sequence of positive numbers satisfying $L_n \to +\infty$, $L_n/\log n \to 0$ $(n \to +\infty)$ and C $(0 < C < +\infty)$ denote some constant not depending on n, but which may assume different values at each appearance.

Let $V_1^2 = \sum_{i=1}^n X_i X'_i$, $A' = (A(T_1), \cdots, A(T_n))$, $V_2^2 = A'A$. Set $Z_{1i} = V_1^{-1}X_i$, $Z_{2i} = V_2^{-1}A(T_i)$, $\theta_1 = V_1(\beta - \beta_0)$, $\theta_2 = V_2(\alpha - \alpha_0)$, $\theta' = (\theta'_1, \theta'_2)$, $Z'_i = (Z'_{1i}, Z'_{2i})$, and set $R_i = g(T_i) - A(T_i)'\alpha_0$ and $D_n = \max_{1\leqslant i\leqslant n}(|R_i| + L_n M_n^{1/2}\|Z_i\|)$. Then

$$\sum_{i=1}^{n} Z_{1i}Z'_{1i} = I_k, \quad \sum_{i=1}^{n} Z_{2i}Z'_{2i} = I_{(m+1)M_n}, \tag{3}$$

$$\hat{\theta}_1 = V_1(\hat{\beta} - \beta_0), \hat{\theta}_2 = V_2(\hat{\alpha} - \alpha_0). \tag{4}$$

and by (2)

$$\begin{aligned}(\hat{\beta}', \hat{\alpha}')' &= \text{Arg}\min_{\beta,\alpha} \sum\nolimits_{i=1}^{n} |g(T_i) - A(T_i)'\alpha_0 + e_i - X'_i(\beta - \beta_0) - A(T_i)'\alpha + A(T_i)'\alpha_0| \\ &= \text{Arg}\min_{\theta_1,\theta_2} \sum\nolimits_{i=1}^{n} |R_i + e_i - Z'_{1i}\theta_1 - Z'_{2i}\theta_2| \\ &= \text{Arg}\min_{\theta} \sum\nolimits_{i=1}^{n} (|R_i + e_i - Z'_i\theta| - |R_i + e_i|).\end{aligned} \tag{5}$$

Lemma 1 Suppose that A3-A5 hold and that $D_n = O_p(1/\log n)$ and $M_n \sim n^{1/(2p+1)}$. Then

$$L_n^{-2}M_n^{-1}\sup_{\|\theta\|\leqslant 1} \left| \sum\nolimits_{i=1}^{n} (|R_i + e_i - L_n M_n^{1/2} Z'_i\theta| - |R_i + e_i|)\right.$$

$$+ L_n M_n^{1/2} \sum_{i=1}^{n} \mathrm{sgn}(e_i) Z'_i \theta - \sum_{i=1}^{n} E_e(| R_i + e_i - L_n M_n^{1/2} Z'_i \theta | - | R_i + e_i |) | = o_p(1),$$

where E_e is the conditional expectation operator given $(X_1, T_1), \cdots, (X_n, T_n)$ and sgn(·) is the sign function.

Proof See Lemma 3.2 in Shi and Li[3].

Lemma 2 Suppose that A1-A5 hold and $M_n \sim n^{1/(2p+1)}$. Then $\| \hat{\theta} \| = O_p(M_n^{1/2})$.

Proof It suffices to show $P\{ \| \hat{\theta} \| < L_n M_n^{1/2} \} \to 1$. We first show that

$$D_n = O_p(1/\log n) = o_p(1), \quad n M_n^{-1} \max_{1 \le i \le n} R_i^2 \le C. \tag{6}$$

Observe that V_2^2 can be denoted by $\mathrm{DIAG}(\boldsymbol{B}_1, \cdots, \boldsymbol{B}_{M_n})$, where $\boldsymbol{B}_v = (b_{vlq})_{(m+1)\times(m+1)}$, $b_{vlq} = \sum_{i=1}^{n} [(T_i - d_v)/h]^{l+q-2} I_{\{|T_i - d_v| \le h\}}$, $l, q = 1, 2, \cdots, m+1$, $v = 1, 2, \cdots, M_n$. Let $\boldsymbol{A}_v = (a_{vlq})_{(m+1)\times(m+1)}$, $a_{vlq} = \int_{|t| \le 1} t^{l+q-2} w(d_v + ht) \mathrm{d}t$, $l, q = 1, 2, \cdots, m+1$, $v = 1, 2, \cdots, M_n$. Since for any $\varepsilon > 0$,

$$\sum_{n=1}^{\infty} P\{| 1/(nh) \sum_{i=1}^{n} [(T_i - d_v)/h]^{l+q-2} I_{\{|T_i - d_v| \le h\}} - \int_{|t| \le 1} t^{l+q-2} w(d_v + ht) \mathrm{d}t | > \varepsilon\}$$

$$\le \sum_{n=1}^{\infty} 1/(\varepsilon^4 n^4) E | \sum_{i=1}^{n} [((T_i - d_v)/h)^{l+q-2} I_{\{|T_i - d_v| \le h\}}/h - \int_{|t| \le 1} t^{l+q-2} w(d_v + ht) \mathrm{d}t] |^4$$

$$\le C \sum_{n=1}^{\infty} (n M_n^4 + n^2 M_n^2)/(\varepsilon^4 n^4) \le C/\varepsilon^4 \sum_{n=1}^{\infty} n^{2/(2p+1)-2} < +\infty, \ (p > 3/2).$$

Then by Borel-Contelli's lemma and note that $1/h = 2M_n$, we have

$$\frac{2M_n}{n} b_{vlq} - a_{vlq} \to 0 \quad a.s., \qquad v = 1, \cdots, M_n, l, q = 1, 2, \cdots, m+1. \tag{7}$$

Let λ_1 denote the smallest eigenvalue of V_1^2 and λ_2 be the smallest eigenvalue of V_2^2, then by condition A1 and A4, according to the argument of Stone[4] and using (7), there is a positive constant λ_0 satisfying $\frac{1}{n}\lambda_1 > \lambda_0$, *a. s.*, $\frac{M_n}{n}\lambda_2 > \lambda_0$ *a. s.*. Hence

$$\begin{aligned} \| Z_i \|^2 &= \| Z_{1i} \|^2 + \| Z_{2i} \|^2 \\ &= X'_i V_1^{-2} X_i + A(T_i)' V_2^{-2} A(T_i) \\ &\le \max_{1 \le i \le n} \| X_i \|^2/(n\lambda_0) + (m+1) M_n/(n\lambda_0) \quad a.s.. \end{aligned} \tag{8}$$

By condition A2, we have

$$\max_{1 \le i \le n} |R_i| = \max_{1 \le i \le n} |g(T_i) - A(T_i)'\alpha_0| \le C M_n^{-p}. \tag{9}$$

So (6) follows from (8), (9) and condition A5 and that $M_n \sim n^{1/(2p+1)}$. Set

$$\Gamma_n(\theta) = \sum_{i=1}^{n} E_e(| R_i + e_i - L_n M_n^{1/2} Z'_i \theta | - | R_i + e_i |).$$

By condition A3, (3) and (6) for θ satisfying $\| \theta \| = 1$ and n sufficiently large, we have

$$\begin{aligned} \Gamma_n(\theta) &= \sum_{i=1}^{n} [2\int_{-(R_i - L_n M_n^{1/2} Z'_i \theta)}^{0} (R_i - L_n M_n^{1/2} Z'_i \theta + x) f(x) \mathrm{d}x - 2\int_{-R_i}^{0} (R_i + x) f(x) \mathrm{d}x] \\ &= \sum_{i=1}^{n} [f(0)(R_i - L_n M_n^{1/2} Z'_i \theta)^2 (1 + o(1)) - f(0) R_i^2 (1 + o(1))] \\ &\ge \sum_{i=1}^{n} [f(0) L_n^2 M_n (Z'_{1i}\theta_1 + Z'_{2i}\theta_2)^2/2 - 4f(0) R_i^2] \\ &\ge f(0) L_n^2 M_n (1 + 2\sum_{i=1}^{n} Z'_{1i}\theta_1 \cdot Z'_{2i}\theta_2)/2 - 4f(0) n \max_{1 \le i \le n} R_i^2. \end{aligned}$$

here $\lim_{n\to\infty} o(1) = 0$ uniformly in $\theta(\|\theta\| = 1)$.

Observe that $2\sum_{i=1}^{n} Z'_{1i}\theta_1 \cdot Z'_{2i}\theta_2 = 2\theta'_1 V_1^{-1} \sum_{i=1}^{n} X_i Z'_{2i}\theta_2$ and

$$\sum_{i=1}^{n} X_i Z'_{2i}\theta_2 = \begin{pmatrix} \sum_{i=1}^{n} X_{i1} Z'_{2i}\theta_2 \\ \vdots \\ \sum_{i=1}^{n} X_{ik} Z'_{2i}\theta_2 \end{pmatrix}.$$

Using $A4$, it follows that $E(\sum_{i=1}^{n} X_{ij}Z'_{2i}\theta_2) = 0$ and that

$$E(\sum_{i=1}^{n} X_{ij}Z'_{2i}\theta_2)^2 = \sum_{i=1}^{n} EX_{ij}^2 E(Z'_{2i}\theta_2)^2 = \sigma_{jj}\|\theta_2\|^2 \leqslant \sigma_{jj},\ j = 1,2,\cdots,k.$$

Hence $\sum_{i=1}^{n} X_i Z'_{2i}\theta_2 = O_p(1)$. Again by $A4$, we have $V_1^2/n \to EXX'$ $a.\ s.$, hence $V_1^{-1} = O_p(n^{-1/2}) = o_p(1)$. Therefore

$$2\theta'_1 V_1^{-1}\sum_{i=1}^{n} X_i Z'_{2i}\theta_2 = o_p(1). \tag{10}$$

uniformly in $\theta(\|\theta\|\leqslant 1)$. Thus by (6) and the above

$$\Gamma_n(\theta) \geqslant f(0)L_n^2 M_n(1+o_p(1))/2. \tag{11}$$

Set

$$G_n(\theta) = \sum_{i=1}^{n}(|R_i+e_i-L_nM_n^{1/2}Z'_i\theta|-|R_i+e_i|) - \sum_{i=1}^{n}E_e(|R_i+e_i-L_nM_n^{1/2}Z'_i\theta|-|R_i+e_i|).$$

It is easy to prove that $M_n^{-1/2}\left\|\sum_{i=1}^{n}\mathrm{sgn}(e_i)Z_i\right\| = O_p(1)$, by (6) and Lemma 1, we deduce that

$$L_n^{-2}M_n^{-1}\sup_{\|\theta\|\leqslant 1}|G_n(\theta)| = o_p(1). \tag{12}$$

From the definition of $\Gamma_n(\theta)$ and $G_n(\theta)$, combining (11) and (12), we have

$$\begin{aligned} &L_n^{-2}M_n^{-1}\inf_{\|\theta\|=1}(\sum_{i=1}^{n}(|R_i+e_i-L_nM_n^{1/2}Z'_i\theta|-|R_i+e_i|)) \\ &\geqslant L_n^{-2}M_n^{-1}\inf_{\|\theta\|=1}\Gamma_n(\theta) - L_n^{-2}M_n^{-1}\sup_{\|\theta\|\leqslant 1}|G_n(\theta)| \geqslant f(0)(1+o_p(1))/2. \end{aligned}$$

So

$$P\{L_n^{-2}M_n^{-1}\inf_{\|\theta\|=1}(\sum_{i=1}^{n}(|R_i+e_i-L_nM_n^{1/2}Z'_i\theta|-|R_i+e_i|))>0\}\to 1.$$

From the convexity of the absolute-valued function $|\cdot|$, we obtain

$$P\{L_n^{-2}M_n^{-1}\inf_{\|\theta\|\geqslant 1}(\sum_{i=1}^{n}(|R_i+e_i-L_nM_n^{1/2}Z'_i\theta|-|R_i+e_i|))>0\}\to 1.$$

Hence

$$P\{\inf_{\|\theta\|\geqslant L_nM_n^{1/2}}(\sum_{i=1}^{n}(|R_i+e_i-Z'_i\theta|-|R_i+e_i|))>0\}\to 1.$$

Therefore

$$P\{\|\hat\theta\| < L_nM_n^{1/2}\}\to 1.$$

Proof of Theorem 1　Let $\eta = \sum_{i=1}^{n}\mathrm{sgn}(e_i)Z_{1i}/(2f(0))$, by (4) in Chen[5] $2f(0)\eta \to_d N(0,I_k)$. By (4) we need only to show that $2f(0)\hat\theta_1 \to_d N(0,I_k)$, so it suffices to prove that for any $\delta>0$, $P\{\|\hat\theta_1-\eta\|<\delta\}\to 1$. So by (5), we need to show that

$$P\{\inf_{\|\theta_1-\eta\|\geqslant\delta}\sum_{i=1}^{n}|R_i+e_i-Z'_{1i}\theta_1-Z'_{2i}\hat\theta_2| > \sum_{i=1}^{n}|R_i+e_i-Z'_{1i}\eta-Z'_{2i}\hat\theta_2|\}\to 1.$$

Using the convexity of the absolute − valued function $|\cdot|$, we need only to prove that

$$P\{\inf_{\|\theta_1-\eta\|=\delta}(\sum_{i=1}^{n}|R_i+e_i-Z'_{1i}\theta_1-Z'_{2i}\hat\theta_2| - \sum_{i=1}^{n}|R_i+e_i-Z'_{1i}\eta-Z'_{2i}\hat\theta_2|)>0\}\to 1.$$

By Lemma 2 and definition of η, it follows that $\|\hat{\theta}_2\| = O_p(M_n^{1/2})$, $\|\eta\| = O_p(1)$, so it suffices to show that for any $L>0, L'>0$

$$P(\{\inf_{\|\theta_1-\eta\|=\delta,\ \|\theta_2\|\le LM_n^{1/2}}(\sum_{i=1}^{n}|R_i+e_i-Z'_{1i}\theta_1-Z'_{2i}\theta_2|-\sum_{i=1}^{n}|R_i+e_i-Z'_{1i}\eta-Z'_{2i}\theta_2|)>0\}\cap\{\|\eta\|\le L'\})\to 1.$$

Set

$$\bar{G}_n(\theta_1,\theta_2)=\sum_{i=1}^{n}(|R_i+e_i-Z'_{1i}\theta_1-Z'_{2i}\theta_2|-|R_i+e_i-Z'_{2i}\theta_2|).$$

Then we need only to prove that

$$P(\{\inf_{\|\theta_1-\eta\|=\delta,\ \|\theta_2\|\le LM_n^{1/2}}(\bar{G}_n(\theta_1,\theta_2)-\bar{G}_n(\eta,\theta_2))>0\}\cap\{\|\eta\|\le L'\})\to 1. \tag{13}$$

Set

$$\bar{\Gamma}_n(\theta_1,\theta_2)=\sum_{i=1}^{n}E_e(|R_i+e_i-Z'_{1i}\theta_1-Z'_{2i}\theta_2|-|R_i+e_i-Z'_{2i}\theta_2|),$$

$$\bar{S}_n(\theta_1,\theta_2)=\bar{G}_n(\theta_1,\theta_2)-\bar{\Gamma}_n(\theta_1,\theta_2)+\sum_{i=1}^{n}\mathrm{sgn}(e_i)Z'_{1i}\theta_1.$$

Then

$$\bar{G}_n(\theta_1,\theta_2)=\bar{\Gamma}_n(\theta_1,\theta_2)-\sum_{i=1}^{n}\mathrm{sgn}(e_i)Z'_{1i}\theta_1+\bar{S}_n(\theta_1,\theta_2).$$

Observe that

$$\begin{aligned}\bar{\Gamma}_n(\theta_1,\theta_2)&=\sum_{i=1}^{n}[f(0)(Z'_{1i}\theta_1)^2-2f(0)Z'_{1i}\theta_1R_i+2f(0)Z'_{1i}\theta_1\cdot Z'_{2i}\theta_2+\tau_{ni}(\theta_1,\theta_2)]\\&=f(0)\|\theta_1\|^2-2f(0)\sum_{i=1}^{n}Z'_{1i}\theta_1R_i+2f(0)\sum_{i=1}^{n}Z'_{1i}\theta_1\cdot Z'_{2i}\theta_2+\sum_{i=1}^{n}\tau_i(\theta_1,\theta_2),\end{aligned}$$

here

$$\begin{aligned}\tau_i(\theta_1,\theta_2)&=2\int_{-(R_i-Z'_{1i}\theta_1-Z'_{2i}\theta_2)}^{0}(R_i+x-Z'_{1i}\theta_1-Z'_{2i}\theta_2)(f(x)-f(0))\mathrm{d}x\\&\quad-2\int_{-(R_i-Z'_{2i}\theta_2)}^{0}(R_i+x-Z'_{2i}\theta_2)(f(x)-f(0))\mathrm{d}x.\end{aligned}$$

Similar to the proof of (10), it follows that

$$2f(0)\sum_{i=1}^{n}Z'_{1i}\theta_1R_i=2f(0)\theta'_1V_1\sum_{i=1}^{n}X_iR_i=O_p(n^{-1/2})\cdot O_p((n\max_{1\le i\le n}R_i^2)^{1/2})=o_p(1) \tag{14}$$

and that

$$2f(0)\sum_{i=1}^{n}Z'_{1i}\theta_1\cdot Z'_{2i}\theta_2=2f(0)\theta'_1V_1\sum_{i=1}^{n}X_iZ'_{2i}\theta_2=O_p(n^{-1/2})\cdot O_p(M_n^{1/2})=o_p(1) \tag{15}$$

uniformly in θ_1 satisfying $\|\theta_1\|\le\tilde{L}$ and θ_2 satisfying $\|\theta_2\|\le LM_n^{1/2}$, where $\tilde{L}=L'+\delta$. By A6, A5, (8) and (9) and the fact that $M_n\sim n^{1/(2p+1)}$, we obtain

$$\begin{aligned}\sum_{i=1}^{n}|\tau_i(\theta_1,\theta_2)|&\le\max_{1\le i\le n}(|R_i|+|Z'_{1i}\theta_1|+|Z'_{2i}\theta_2|)\sum_{i=1}^{n}(|R_i|+|Z'_{1i}\theta_1|+|Z'_{2i}\theta_2|)^2\\&\le CL^2M_n\max_{1\le i\le n}(|R_i|+|Z'_{1i}\theta_1|+|Z'_{2i}\theta_2|)=o_p(1)\end{aligned}$$

uniformly in θ_1 ($\|\theta_1\|\le\tilde{L}$) and θ_2 ($\|\theta_2\|\le LM_n^{1/2}$). Hence

$$\bar{\Gamma}_n(\theta_1,\theta_2)=f(0)\|\theta_1\|^2+o_p(1). \tag{16}$$

Thus

$$\bar{G}_n(\theta_1,\theta_2)=f(0)\|\theta_1\|^2-2f(0)\eta'\theta_1+\bar{S}_n(\theta_1,\theta_2)+o_p(1), \tag{17}$$

uniformly in θ_1 ($\|\theta_1\|\le\tilde{L}$) and θ_2 ($\|\theta_2\|\le LM_n^{1/2}$). Using the fact that $2\eta'\theta_1=\|\eta\|^2+\|\theta_1\|^2-\|\eta-\theta_1\|^2$ and that $\|\theta_1-\eta\|=\delta$, we have

$$\bar{G}_n(\theta_1,\theta_2)=f(0)\delta^2-f(0)\|\eta\|^2+\bar{S}_n(\theta_1,\theta_2)+o_p(1). \tag{18}$$

By (17)

$$\bar{G}_n(\eta,\theta_2)=-f(0)\|\eta\|^2+\bar{S}_n(\eta,\theta_2)+o_p(1), \tag{19}$$

it follows from (18) and (19) that

$$\bar{G}_n(\theta_1,\theta_2)\geqslant f(0)\delta^2+\bar{G}_n(\eta,\theta_2)-2\sup_{\|\theta_1\|\leqslant L,\ \|\theta_2\|\leqslant LM_n^{1/2}}|\bar{S}_n(\theta_1,\theta_2)|+o_p(1).$$

By A5, using the method similar to the method of Lemma 1 and noting that $p>3/2$, it can be shown that $\sup_{\|\theta_1\|\leqslant L,\ \|\theta_2\|\leqslant LM_n^{1/2}}|\bar{S}_n(\theta_1,\theta_2)|=o_p(1)$. Hence

$$P(\{\inf_{\|\theta_1-\eta\|=\delta,\ \|\theta_2\|\leqslant LM_n^{1/2}}(\bar{G}_n(\theta_1,\theta_2)-\bar{G}_n(\eta,\theta_2))>0\}\cap\{\|\eta\|\leqslant L'\})\to 1.$$

According to (13), theorem 1 follows.

Proof of Theorem 2 Let $\bar{X}=\frac{1}{n}\sum_{i=1}^{n}X_i$, $\bar{X}_j=\frac{1}{n}\sum_{i=1}^{n}X_{ij}, j=1,\cdots,k$, then

$$Y_i=(X_i-\bar{X})'\beta_0+g(T_i)+\bar{X}'\beta_0+e_i.$$

Set $X_{ni}=X_i-\bar{X}$, $g_1(t)=g_1(t,\bar{X})=g(t)+\bar{X}'\beta_0$, then X_{ni} satisfies A4 and A5 and $g_1(t,\bar{X})$ satisfies A2 and

$$Y_i=X'_{ni}\beta_0+g_1(T_i,\bar{X})+e_i. \tag{20}$$

The proof of theorem 2 is similar to the proof of theorem 1, in fact if (10), (14) and (15) hold, then theorem 1 holds for model (20). So we only show that (10), (14) and (15) hold. Since $E(\sum_{i=1}^{n}X_{nij}Z'_{2i}\theta_2)=0$, and

$$\begin{aligned}E(\sum_{i=1}^{n}X_{nij}Z'_{2i}\theta_2)^2&=\sum_{i=1}^{n}\sum_{l=1}^{n}E(X_{ij}-\bar{X}_j)(X_{lj}-\bar{X}_j)E(Z'_{2i}\theta_2\cdot Z'_{2l}\theta_2)\\&=\bar{\sigma}_{jj}\sum_{i=1}^{n}E(Z'_{2i}\theta_2)^2-(\bar{\sigma}_{jj}/n)\sum_{i=1}^{n}\sum_{l=1}^{n}E(Z'_{2i}\theta_2\cdot Z'_{2l}\theta_2)\\&\leqslant 2\|\theta_2\|^2\bar{\sigma}_{jj}\leqslant 2\bar{\sigma}_{jj}\quad j=1,\cdots,k.\end{aligned}$$

Therefore $\sum_{i=1}^{n}X_{ni}Z'_{2i}\theta_2=O_p(1)$. By A4', we have $V_1^{-1}=O_p(n^{-1/2})=o_p(1)$. Therefore $2\theta'_1V_1^{-1}\sum_{i=1}^{n}X_{ni}Z'_{2i}\theta_2=o_p(1)$ uniformly in $\theta(\|\theta\|\leqslant 1)$. Thus (10) follows. Similar to the proof of (10), we can prove (14) and (15). So theorem 2 follows.

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