第 29 卷第 4 期 南京师大学报(自然科学版) Vol. 29 No. 4
2006 年 12 月 JOURNAL OF NANJING NORMAL UNIVERSITY(Natural Science) Dec,2006

Radiative Weak Annihilation $B \to D_{(s)}^{*} \gamma$ Decays

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Abstract: Starting from the low energy effective Hamiltonian for W-annihilation, we derive the effective Lagrangian for weak annihilation bremsstrahlung processes at the quark level and then apply it to the radiative weak annihilation decays $B \to D_{(s)}^{*} \gamma$. Comparisons with other predictions are also given. Our results indicate that these decays should be detected at B-factories and LHC.

Key words: effective Hamiltonian, effective Lagrangian, W-annihilation, decay

CLC number O572.24 **Document code** A **Article ID** 1001-4616(2006)04-0041-05

辐射弱衰变 $B \to D_{(s)}^{*} \gamma$ 的研究

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[摘要] 从 W 湮灭的低能有效哈米顿出发,在夸克层次对弱湮灭辐射跃迁过程导出了有效拉格朗日,然后用于计算弱辐射湮灭过程 $B \to D_{(s)}^{*} \gamma$,并且与其他模型预测进行了对比,结果表明这些衰变道可在近期的 B 工厂和 LHC 上被探测到.

[关键词] 有效哈密顿, 有效拉格朗日, W 湮灭, 衰变

0 Introduction

In the forthcoming years, more and more data of B decay processes will be available at Tevatron, CLEO and the B-factories, more and more ever regarded rare decays of B meson with branching ratios of 10^{-7} or 10^{-8} are possible to be measured, bring the necessity of estimating these decay processes in advance. In this paper, we focus on the radiative weak annihilation processes $B \to D_s^* \gamma$ and $B \to D^* \gamma$, collectively $B \to D_{(s)}^{*} \gamma$ decays.

These decays were first studied as a probe of V_{ub} in Ref. [1] and the branching ratios were estimated to be 4×10^{-5} for $B \to D_s^* \gamma$ and 2×10^{-6} for $B \to D^* \gamma$. At the same time, as two radiative weak annihilation processes, they were also investigated in Ref. [2] and the calculated branching ratios are 1×10^{-7} and 7×10^{-9} for $B \to D_s^* \gamma$ and $B \to D^* \gamma$, respectively. There exists vast differences between these two predictions. To clarify which is of the correct order, we argue that the effective Lagrangian method developed by Cheng[3], which works well when both initial and final hadrons contain a heavy quark, is applicable to these processes and use the factorization hypothesi[4], which is known to work well for nonleptonic weak decays of heavy mesons, to evaluate the hadronic matrix elements and decay amplitudes. Our results favor the later.

1 Local Effective Hamiltonian for $B \to D_{(s)}^{*} \gamma$ Decays

At the lowest level, these decay are induced only by weak W-annihilation diagram (Fig. 1) with the emitted photon coupling to any of the charged lines. Because the diagram with the photon emitted from the W-boson is

Received date: 2006-06-20.

Foundation item: Supported by the National Natural Science Foundation of China(10275035) and the Natural Science Foundation of Education Department of Jiangsu Province(2003102TSJB137).

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suppressed by a factor G_F, we consider only the case with the photon emitted from quark lines.

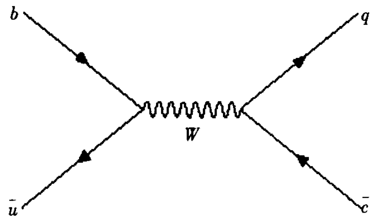

Fig.1 W-annihilation diagram

The QCD corrected effective weak Hamiltonian for W-annihilation processes $b\bar{u}\to q\bar{c}(q=d,s)$ is given by

$$\mathscr{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ub} V_{cq}^{*} (C_1 O_1 + C_2 O_2) \tag{1}$$

with

$$O_1 = (\bar{u}b)_{V-A}(\bar{q}c)_{V-A},\ O_2 = (\bar{u}c)_{V-A}(\bar{q}b)_{V-A} \tag{2}$$

At the scale $\mu = m_b$, the renormalization scheme independent Wilson coefficient functions C_1 and C_2 take[5,6]

$$C_1 = 1.150\,2,\quad C_2 = -0.312\,5 \tag{3}$$

For the photon emission process $b\bar{u}\to q\bar{c}\gamma$, the amplitudes mediated by the operator O_1 (Fig. 2) are (up to a factor $\frac{G_F}{\sqrt{2}} V_{ub} V_{cq}^{*} C_1$)

$$\begin{aligned}
\mathscr{A}_1^a &= \bar{u}_q(-iee_q\gamma^{\mu})\frac{i}{\not{p}_q+\not{k}-m_q}\gamma^{v}(1-\gamma_5)v_c\bar{v}_u\gamma_v(1-\gamma_5)u_b\varepsilon_{\mu} \\
&= ee_q\bar{u}_q\gamma^{\mu}\frac{1}{\not{p}_q+\not{k}-m_q}\gamma^{v}(1-\gamma_5)v_c\bar{v}_u\gamma_v(1-\gamma_5)u_b\varepsilon_{\mu} \\
\mathscr{A}_1^b &= ee_b\bar{u}_q\gamma^{v}(1-\gamma_5)v_c\bar{v}_u\gamma_v(1-\gamma_5)\frac{1}{\not{p}_b-\not{k}-m_b}\gamma^{\mu}u_b\varepsilon_{\mu} \\
\mathscr{A}_1^c &= ee_u\bar{u}_q\gamma^{v}(1-\gamma_5)v_c\bar{v}_u\gamma^{\mu}\frac{1}{-\not{p}_u+\not{k}-m_u}\gamma_v(1-\gamma_5)u_b\varepsilon_{\mu} \\
\mathscr{A}_1^d &= ee_c\bar{u}_q\gamma^{v}(1-\gamma_5)\frac{1}{-\not{p}_c-\not{k}-m_c}\gamma^{\mu}v_c\bar{v}_u\gamma_v(1-\gamma_5)u_b\varepsilon_{\mu}
\end{aligned} \tag{4}$$

where k and ε are the photon momentum and polarization vector, respectively. Similarly the amplitudes mediated by the operator O_2 (Fig. 3) are (up to a factor $\frac{G_F}{\sqrt{2}} V_{ub} V_{cq}^{*} C_2$)

$$\begin{aligned}
\mathscr{A}_2^a &= ee_q\bar{u}_q\gamma^{\mu}\frac{1}{\not{p}_q+\not{k}-m_q}\gamma^{v}(1-\gamma_5)u_b\bar{v}_u\gamma_v(1-\gamma_5)v_c\varepsilon_{\mu} \\
\mathscr{A}_2^b &= ee_b\bar{u}_q\gamma^{v}(1-\gamma_5)\frac{1}{\not{p}_b-\not{k}-m_b}\gamma^{\mu}u_b\bar{v}_u\gamma_v(1-\gamma_5)v_c\varepsilon_{\mu} \\
\mathscr{A}_2^c &= ee_u\bar{u}_q\gamma^{v}(1-\gamma_5)u_b\bar{v}_u\gamma^{\mu}\frac{1}{-\not{p}_u+\not{k}-m_u}\gamma_v(1-\gamma_5)v_c\varepsilon_{\mu} \\
\mathscr{A}_2^d &= ee_c\bar{u}_q\gamma^{v}(1-\gamma_5)u_b\bar{v}_u\gamma_v(1-\gamma_5)\frac{1}{-\not{p}_c-\not{k}-m_c}\gamma^{\mu}v_c\varepsilon_{\mu}
\end{aligned} \tag{5}$$

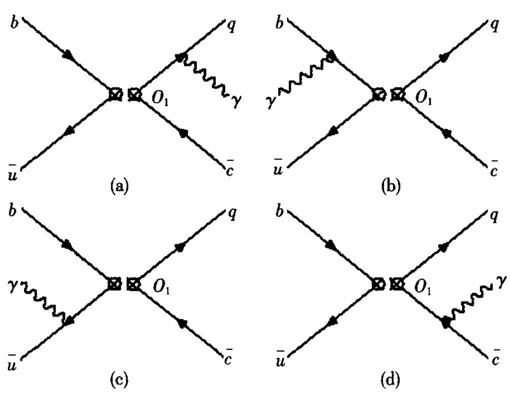

Fig.2 W-annihilation diagram via O_1

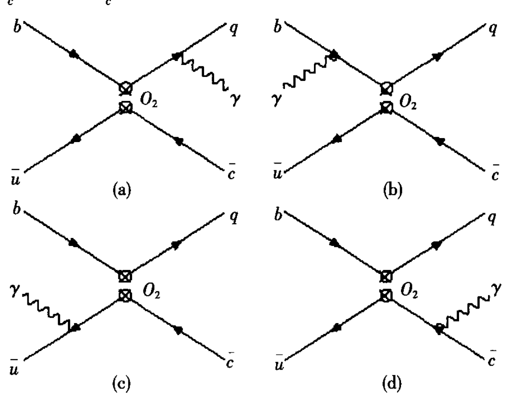

Fig.3 W-annihilation diagram via O_2

If we parameterize the quark momentum in terms of their velocities as done in Ref. [3]

$$p_b = m_b v, p_u = m_u v, p_q = m_q v', p_c = m_c v' \tag{6}$$

here the light quark mass appearing in (4), (5) and (6) are of the constituent type. Obviously, this parameterization (6) does not provide a complete description of the Fermi motion inside the bound state but just average effects, however as will see later, it does eliminate the photon's coupling to the convection currents and makes the effective interaction local and gauge invariant. Therefore, it greatly simplifies the calculation.

With the parameterization given in (6), we find that the contributions from the convectional currents add up to zero, and the amplitude arises entirely from the magnetic moments of the quarks. Then we have

$$\begin{aligned}
\mathscr{A}_1 &= \mathscr{A}_1^a + \mathscr{A}_1^b + \mathscr{A}_1^c + \mathscr{A}_1^d \\
&= -\frac{ie}{m_i^2 - m_f^2}\Big[e_q \frac{m_f}{m_q}\bar{u}_q \sigma^{\mu\lambda} k_\lambda \gamma^v (1-\gamma_5) v_c \bar{v}_u \gamma_v (1-\gamma_5) u_b \varepsilon_\mu \\
&\quad - e_b \frac{m_i}{m_b}\bar{u}_q \gamma^v (1-\gamma_5) v_c \bar{v}_u \gamma_v (1-\gamma_5) \sigma^{\mu\lambda} k_\lambda u_b \varepsilon_\mu \\
&\quad - e_u \frac{m_i}{m_u}\bar{u}_q \gamma^v (1-\gamma_5) v_c \bar{v}_u \sigma^{\mu\lambda} k_\lambda \gamma_v (1-\gamma_5) u_b \varepsilon_\mu \\
&\quad + e_c \frac{m_f}{m_c}\bar{u}_q \gamma^v (1-\gamma_5) \sigma^{\mu\lambda} k_\lambda v_c \bar{v}_u \gamma_v (1-\gamma_5) u_b \varepsilon_\mu \Big]
\end{aligned} \tag{7}$$

$$\begin{aligned}
\mathscr{A}_2 &= \mathscr{A}_2^a + \mathscr{A}_2^b + \mathscr{A}_2^c + \mathscr{A}_2^d \\
&= -\frac{ie}{m_i^2 - m_f^2}\Big[e_q \frac{m_f}{m_q}\bar{u}_q \sigma^{\mu\lambda} k_\lambda \gamma^v (1-\gamma_5) u_b \bar{v}_u \gamma_v (1-\gamma_5) u_c \varepsilon_\mu \\
&\quad - e_b \frac{m_i}{m_b}\bar{u}_q \gamma^v (1-\gamma_5) \sigma^{\mu\lambda} k_\lambda u_b \bar{v}_u \gamma_v (1-\gamma_5) v_c \varepsilon_\mu \\
&\quad - e_u \frac{m_i}{m_u}\bar{u}_q \gamma^v (1-\gamma_5) u_b \bar{v}_u \sigma^{\mu\lambda} k_\lambda \gamma_v (1-\gamma_5) v_c \varepsilon_\mu \\
&\quad + e_c \frac{m_f}{m_c}\bar{u}_q \gamma^v (1-\gamma_5) u_b \bar{v}_u \gamma_v (1-\gamma_5) \sigma^{\mu\lambda} k_\lambda v_c \varepsilon_\mu \Big]
\end{aligned} \tag{8}$$

where $m_i = m_b + m_u$ and $m_f = m_c + m_q$. After Dirac algebra operation, the results can be simplified as

$$\begin{aligned}
\mathscr{A}_1 &= \frac{e}{m_i^2 - m_f^2}\Big\{\Big(e_q \frac{m_f}{m_q} + e_u \frac{m_i}{m_u}\Big)(\tilde{F}_{\mu v} + iF_{\mu v}) + \Big(e_c \frac{m_f}{m_c} + e_b \frac{m_i}{m_b}\Big)(\tilde{F}_{\mu v} - iF_{\mu v})\Big\} \\
&\quad \bar{u}_q \gamma^\mu (1-\gamma_5) v_c \bar{v}_u \gamma^v (1-\gamma_5) u_b \\
\mathscr{A}_2 &= \frac{e}{m_i^2 - m_f^2}\Big\{\Big(e_q \frac{m_f}{m_q} + e_u \frac{m_i}{m_u}\Big)(\tilde{F}_{\mu v} + iF_{\mu v}) - \Big(e_c \frac{m_f}{m_c} + e_b \frac{m_i}{m_b}\Big)(\tilde{F}_{\mu v} - iF_{\mu v})\Big\} \\
&\quad \bar{u}_q \gamma^\mu (1-\gamma_5) u_b \bar{v}_u \gamma^v (1-\gamma_5) v_c
\end{aligned} \tag{9}$$

From the above discussions, we can find that the effective Hamiltonian for photon emission process $b\bar{u} \to q\bar{c}\gamma$ can be written as

$$\mathscr{H}_{\text{eff}}(b\bar{u} \to q\bar{c}\gamma) = \frac{G_F}{\sqrt{2}} V_{ub} V_{cq}^* (C_1 Q_1 + C_2 Q_2) \tag{10}$$

with

$$\begin{aligned}
Q_{1,2} &= \frac{e}{m_i^2 - m_f^2}\Big\{\tilde{F}_{\mu v}\Big[\Big(e_q \frac{m_f}{m_q} + e_u \frac{m_i}{m_u}\Big) \pm \Big(e_c \frac{m_f}{m_c} + e_b \frac{m_i}{m_b}\Big)\Big] \\
&\quad + iF_{\mu v}\Big[\Big(e_q \frac{m_f}{m_q} + e_u \frac{m_i}{m_u}\Big) \mp \Big(e_c \frac{m_f}{m_c} + e_b \frac{m_i}{m_b}\Big)\Big]\Big\} O_{1,2}^{\mu v}
\end{aligned} \tag{11}$$

where

$$O_1^{\mu v} = \bar{q}\gamma^\mu (1-\gamma_5) c \bar{u} \gamma^v (1-\gamma_5) b$$

$$O_2^{\mu v} = \bar{q}\gamma^\mu (1-\gamma_5) b \bar{u} \gamma^v (1-\gamma_5) c$$

$$\widetilde{F}_{\mu\nu} \equiv \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}F^{\alpha\beta} \tag{12}$$

2 Decay Amplitudes

To calculate the hadronic matrix elements and the decay amplitudes, we use the factorization method[4]. Then the decay amplitudes can be written as

$$\begin{aligned}\mathscr{A}(B \to D^*_{(s)}\gamma) &= < D^*_{(s)} \mid \mathscr{H}_{\text{eff}} \mid B > \\ &= \frac{G_F}{\sqrt{2}}V_{ub}V^*_{cq}\left(C_1 + \frac{1}{N_c}C_2\right)\frac{e}{m_i^2 - m_f^2} < D^*_{(s)} \mid \bar{q}\gamma_\mu(1-\gamma_5)c \mid 0 > < 0 \mid \bar{u}\gamma_\nu(1-\gamma_5)b \mid B > \\ &\times \left\{\widetilde{F}^{\mu\nu}\left[\left(\frac{e_c}{m_c} + \frac{e_q}{m_q}\right)m_f + \left(\frac{e_u}{m_u} + \frac{e_b}{m_b}\right)m_i\right] + iF^{\mu\nu}\left[\left(\frac{e_c}{m_c} - \frac{e_q}{m_q}\right)m_f + \left(\frac{e_u}{m_u} - \frac{e_b}{m_b}\right)m_i\right]\right\}\end{aligned} \tag{13}$$

Using the expression for one-body matrix elements

$$< 0 \mid A_\mu \mid P(p) > = if_p p_\mu, \qquad < 0 \mid V_\mu \mid V(p,\eta) > = if_V m_V \eta_\mu \tag{14}$$

therefore

$$< D^*_{(s)} \mid \bar{q}\gamma_\mu(1-\gamma_5)c \mid 0 > < 0 \mid \bar{u}\gamma_\nu(1-\gamma_5)b \mid B(p) > = -f_B f_{D^*_{(s)}} M_{D^*_{(s)}}\eta_\mu p_\nu \tag{15}$$

then the decay amplitudes can be written as the following gauge invariant form

$$\mathscr{A}[B(p) \to D^*_{(s)}(q,\eta)\gamma(k,\varepsilon)] = i\varepsilon_{\mu\nu\alpha\beta}\varepsilon^\mu\eta^\nu p^\alpha k^\beta f_1(k^2) + (\varepsilon\cdot\eta p\cdot k - \varepsilon\cdot k\eta\cdot p)f_2(k^2) \tag{16}$$

with

$$\begin{aligned}f_1 &= K\left[\left(\frac{e_c}{m_c} + \frac{e_q}{m_q}\right)\frac{M_{D^*_{(s)}}}{m_B} + \left(\frac{e_u}{m_u} + \frac{e_b}{m_b}\right)\right] \\ f_2 &= K\left[\left(\frac{e_c}{m_c} - \frac{e_q}{m_q}\right)\frac{M_{D^*_{(s)}}}{m_B} - \left(\frac{e_u}{m_u} - \frac{e_b}{m_b}\right)\right]\end{aligned} \tag{17}$$

where

$$K = \frac{eG_F}{\sqrt{2}}\frac{m_B m_{D^*_{(s)}}}{m_B^2 - m_{D^*_{(s)}}}\left(C_1 + \frac{1}{N_c}C_2\right)V_{ub}V^*_{cq}f_B f_{D^*_{(s)}} \tag{18}$$

3 Numerical Results and Discussion

The decay width implied by the amplitude (16) in the B rest framework is given by

$$\Gamma(B \to D^*_{(s)}\gamma) = \frac{1}{32\pi}\frac{(m_B^2 - m^2_{D^*_{(s)}})^3}{m_B^3}(\mid f_1\mid^2 + \mid f_2\mid^2) \tag{19}$$

In order to get the numerical results, we adopt the following parameters

(1) Meson masses (GeV)[7]

$$m_B = 5.279, \quad m_{D_s^*} = 2.11, \quad m_{D^*} = 2.01$$

(2) Constituent quark masses[3]

$$m_u = 338\ \text{MeV}, m_d = 322\ \text{MeV}, m_s = 510\ \text{MeV}, m_c = 1.6\ \text{GeV}, m_b = 5\ \text{GeV}$$

(3) Decay constants of mesons (GeV)

$$f_B = 170, \quad f_{D_s^*} = 200, \quad f_{D^*} = 200$$

(4) CKM matrix elements

$$V_{ub} = 3.08\times10^{-3}, \quad v_{cd} = 0.22, \quad V_{cs} = 0.9745$$

where V_{ub} is chosen from $\mid V_{ub}V_{cs}\mid \sim 3\times10^{-3}$ as done in Ref. [1].

(5) Choices of N_c

In QCD theory, $N_c = 3$. Because we use the factorization method and do not take into account the non-factorization effects, here N_c is regarded as a phenomenological parameter and takes three different values $N_c = 2, 3, \infty$ as done in hadronic B meson decays[8-10].

(6) Lifetime of B meson[7]

$$\tau(B^-) = 1.65 \times 10^{-12}\text{s}$$

Our numerical results are collected in Table 1. For comparisons, we also give the predictions available.

Table 1 Branching ratios calculated by Eq. 19

Decays	$N_c = 2$	$N_c = 3$	$N_c = \infty$	Ref. [1]	Ref. [2]
$B \to D^* \gamma$	7.84×10^{-9}	8.68×10^{-9}	1.08×10^{-8}	2×10^{-6}	7×10^{-9}
$B \to D_s^* \gamma$	1.96×10^{-7}	2.17×10^{-7}	2.62×10^{-7}	4×10^{-5}	1×10^{-7}

From our numerical results we can see that, no matter the value of N_c, the estimated branching ratios are of the order 10^{-8} and 10^{-7} for $B \to D^* \gamma$ and $B \to D_s^* \gamma$, respectively, which agree with Ref. [2] despite a very different calculation method, and are very different from Ref. [1]. Therefore we can conclude that the results of Ref. [2] may be of the right order. As discussed in Ref. [2], these decays may be detected at the B-factories and Large Hadronic Collider (LHC), we wait the experimental observation. Once observed, these decays will present an interesting probe for CKM matrix elements V_{ub} as discussed in Ref. [1].

In conclusion, using the effective Lagrangian method and the factorization hypothesis, we evaluate the decay branching ratios $B \to D_{(s)}^* \gamma$ and confirm the results of Ref. [2] in a totally different method.

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