一类非线性时滞系统的鲁棒H_∞几乎干扰解耦

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[摘要] 研究了一类非线性时滞系统的鲁棒H_∞控制问题. 基于 Lyapunov函数递归设计方法, 构造了一种新的控制律和H_∞控制器. 通过技巧性地构造正定 Lyapunov函数, 解决了时滞问题, 实现了递归设计的推广. 数值例子和仿真说明了结论的正确性.

[关键词] 非线性系统, 时滞, 鲁棒H_∞控制, 递归
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Robust H_∞ Almost Disturbance Decoupling for a Class of Time-Delay Nonlinear Systems

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Abstract Robust H_∞ control problem for a class of time-delay nonlinear systems is discussed. A recursive Lyapunov-based design approach is developed to construct a new control law and H_∞ controller. By choosing ideal Lyapunov functionals skillfully, the time-delay term of nonlinear systems is dealed with. Furthermore Backstepping method is extended. Finally, illustrative example and simulation results verify the correctness of the conclusion.

Key words nonlinear systems, time-delay, robust H_∞ control, Backstepping

非线性控制系统的设计是控制领域中最前沿和最具挑战性的课题之一, 也是近年来控制理论研究的重点和热点之一, 所以深入地研究非线性系统的理论和方法具有重要的实际意义[1-6]. 文献[3]研究了一类具有不确定界的非线性时滞系统的自适应控制问题; 文献[4]解决了一类不确定非线性时滞系统的输出反馈镇定问题, 但都未考虑非线性时滞系统的鲁棒H_∞控制问题.

本文研究了一类具时滞的非线性系统, 基于 Lyapunov函数的递推设计方法, 设计了新的控制律和H_∞控制器, 本质改进了 Backstepping方法.

系统描述:

考虑不确定时滞非线性系统

$$\begin{cases} \dot{x}_i(t) = x_{i+1}(t) + \phi_i(t, y(t), y(t-d(t))) + \varphi_i(\boldsymbol{x}_i(t))\omega, \\ \dot{x}_n(t) = u(t) + \phi_n(t, y(t), y(t-d(t))) + \varphi_n(\boldsymbol{x}_n(t))\omega, \\ y(t) = h(x_1), \quad i = 1, 2, \cdots, n-1, \end{cases} \tag{1}$$

其中$\boldsymbol{x}_i = [x_1, \cdots, x_i]^{\mathrm{T}}$, $x_i(t) \in \mathbf{R}$, $u(t) \in \mathbf{R}$, $y(t) \in \mathbf{R}$分别表示系统状态、控制输入和系统输出, $\omega \in \mathbf{R}$为干扰信号, $d(t)$为时滞且满足 $0 \leqslant \dot{d}(t) \leqslant \eta < 1$, $\varphi_i(\cdot)$和$\phi_i(\cdot)$均为光滑函数, $h(x_1)$为已知光滑函数且$h(0) = 0$, 则存在非负光滑函数$\tau_1(x_1)$满足$h(x_1) = x_1(t)\tau_1(x_1)$.

假设 存在非负常数a_i、b_i, 使得 $|\phi_i(t, y(t), y(t-d(t)))| \leqslant a_i|y(t)| + b_i|y(t-d(t))|$.

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引理 1[1] (Young不等式) 对任意 $x, y \in \mathbf{R}$ 有

$$xy \leqslant \frac{\varepsilon^p}{p}|x|^p + \frac{1}{q\varepsilon^q}|y|^q,$$

其中 $\varepsilon > 0$, 常数 $p > 1$ 和 $q > 1$ 满足 $(p-1)(q-1) = 1$.

1 主要结果

作坐标变换

$$z_1(t) = x_1(t),\ z_i(t) = x_i(t) + \alpha_{i-1}(\boldsymbol{x}_{i-1}(t)),\ i = 2, \cdots, n, \tag{2}$$

α_{i-1} 是待定的. 于是系统 (1) 可变形为

$$\begin{aligned}
\dot z_1(t) &= z_2(t) - \alpha_1 + \phi_1(t, y(t), y(t-d(t))) + \varphi_1(x_1(t))\omega,\\
\dot z_i(t) &= z_{i+1}(t) - \alpha_i + \phi_i(t, y(t), y(t-d(t))) + \varphi_i(\boldsymbol{x}_i(t))\omega +\\
&\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}(z_{j+1}(t) - \alpha_j + \varphi_j(t, y(t), y(t-d(t))) + \varphi_j(\boldsymbol{x}_j(t))\omega),\\
\dot z_n(t) &= u(t) + \phi_n(t, y(t), y(t-d(t))) + \varphi_n(\boldsymbol{x}_n(t))\omega +\\
&\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}(z_{j+1}(t) - \alpha_j + \varphi_j(t, y(t), y(t-d(t))) + \varphi_j(\boldsymbol{x}_j(t))\omega).
\end{aligned} \tag{3}$$

定理 对任意给定的正数 $\gamma < 1$, 设计光滑鲁棒动态控制器

$$u = u(x, t),\ u(0, t) = 0,$$

可使得闭环系统 (1) 满足

(i) 对任意 $\omega(t) \in L_{2m}$, 系统初始状态 $x(0) = 0$ 的输出响应 $y(\cdot)$ 满足

$$\int_0^t |y(s)|^{2m}\,\mathrm{d}s \leqslant \gamma^2 \int_0^t \|\omega\|^{2m}\,\mathrm{d}s,\quad \forall t \geqslant 0,$$

$$L_p = \{Z(t) \mid \int_0^\infty \|Z(t)\|^p\,\mathrm{d}s < \infty,\ p \geqslant 1\}.$$

(ii) 当 $\omega(t) = 0\ (t \geqslant 0)$ 时, 闭环系统在平衡点 $x = 0$ 全局渐近稳定.

证明 第 1步: 取正定 Lyapunov函数

$$V_1(z_1) = \frac{1}{2}z_1^2(t) + \frac{\varepsilon}{2(1-\eta)}\int_{d(t)}^t y^2(s)\,\mathrm{d}s, \tag{4}$$

由引理得

$$\dot V_1 + y^{2m}(t) - \beta\|\omega\|^{2m} \leqslant z_1(t)(z_2(t) - g(z_1(t))) - K_1 z_1^2(t) + \frac{\varepsilon}{2}y^2(t)\left(1 + \frac{1}{1-\eta}\right), \tag{5}$$

取

$$\alpha_1 = \frac{1}{2\varepsilon}z_1(t)(a_1^2 + b_1^2) + \varphi_1\left(1 - \frac{1}{2m}\right)\left(\frac{z_1\varphi_1}{2m\beta}\right)^{\frac{1}{2m-1}} + g(z_1(t)) + K_1 z_1(t) + \tau_1^{2m} z_1^{2m-1}(t), \tag{6}$$

其中 K_1 及后面的 $K_i\ (2 \leqslant i \leqslant n)$ 均为选定的正常数, $g(z_1)$ 待定.

第 k 步: 假设在第 k 步存在 α_k 及正定 Lyapunov函数

$$\begin{aligned}
\alpha_k &= z_{k-1}(t) + \frac{z_k(t)}{2\varepsilon}(a_k^2 + b_k^2) + \sum_{i=1}^{k-1}\frac{\partial\alpha_{k-1}}{\partial x_i}(z_{i+1}(t) - \alpha_i) +\\
&\sum_{i=1}^{k-1}\frac{z_k(t)}{2\varepsilon}\left(\frac{\partial\alpha_{k-1}}{\partial x_i}\right)^2(a_i^2 + b_i^2) + \Delta_k\left(1 - \frac{1}{2m}\right)\left(\frac{z_k\Delta_k}{2m\beta}\right)^{\frac{1}{2m-1}} + K_k z_k(t),
\end{aligned} \tag{7}$$

其中 $\Delta_k = \varphi_k + \sum_{i=1}^{k-1}\frac{\partial\alpha_{k-1}}{\partial x_i}\varphi_i$,

$$V_k(z_1, \cdots, z_k) = V_{k-1} + \frac{1}{2}z_k^2(t) + \frac{k\varepsilon}{2(1-\eta)}\int_{d(t)}^t y^2(s)\,\mathrm{d}s \tag{8}$$

满足

$$\dot V_k + y^{2m}(t) - k\beta\|\omega\|^{2m} \leqslant z_k(t)z_{k+1}(t) - z_1(t)g(z_1(t)) - \sum_{i=1}^{k}K_i z_i^2(t) +$$

$$\frac{k(k+1)\varepsilon}{4}y^2(t)\left(1+\frac{1}{1-\eta}\right), \tag{9}$$

则对第 $k+1$ 步,考虑正定 Lyapunov函数

$$V_{k+1}(z_1, \cdots, z_{k+1}) = V_k + \frac{1}{2}z_{k+1}^2(t) + \frac{(k+1)\varepsilon}{2(1-\eta)}\int_{t-d(t)}^{t} y^2(s)\,\mathrm{d}s, \tag{10}$$

$$\dot{V}_{k+1} + y^{2m}(t) - (k+1)\beta\|\omega\|^{2m} \leqslant$$

$$z_k(t)z_{k+1}(t) - z_1(t)g(z_1(t)) - \sum_{i=1}^{k}K_i z_i^2(t) + \frac{k(k+1)\varepsilon}{4}y^2(t)\left(1+\frac{1}{1-\eta}\right) +$$

$$z_{k+1}(t)(z_{k+2}(t) - \alpha_{k+1} + \phi_{k+1}(t, y(t), y(t-d(t))) + \varphi_{k+1}(\boldsymbol{x}_{k+1}(t))\omega +$$

$$\sum_{i=1}^{k}\frac{\partial\alpha_k}{\partial x_i}(z_{i+1}(t) - \alpha_i + \phi_i(t, y(t), y(t-d(t))) + \varphi_i(\boldsymbol{x}_i(t))\omega)) - \beta\|\omega\|^{2m} +$$

$$\frac{(k+1)\varepsilon}{2(1-\eta)}(y^2(t) - (1-\eta)y^2(t-d(t))) \leqslant$$

$$z_k(t)z_{k+1}(t) - z_1(t)g(z_1(t)) - \sum_{i=1}^{k}K_i z_i^2(t) + z_{k+1}(t)\left(z_{k+2}(t) - \alpha_{k+1} + \right.$$

$$\frac{z_{k+1}(t)}{2\varepsilon}(a_{k+1}^2 + b_{k+1}^2) + \sum_{i=1}^{k}\frac{\partial\alpha_k}{\partial x_i}(z_{i+1}(t) - \alpha_i) + \sum_{i=1}^{k}\frac{z_{k+1}(t)}{2\varepsilon}\times$$

$$\left.\left(\frac{\partial\alpha_k}{\partial x_i}\right)^2(a_i^2 + b_i^2) + \Delta_k\left(1-\frac{1}{2m}\right)\left(\frac{z_k\Delta_k}{2m\beta}\right)^{\frac{1}{2m-1}}\right) + \frac{(k+1)(k+2)\varepsilon}{4}y^2(t)\left(1+\frac{1}{1-\eta}\right), \tag{11}$$

$$\alpha_{k+1} = z_k(t) + \frac{z_{k+1}(t)}{2\varepsilon}(a_{k+1}^2 + b_{k+1}^2) + \sum_{i=1}^{k}\frac{\partial\alpha_k}{\partial x_i}(z_{i+1}(t) - \alpha_i) + \sum_{i=1}^{k}\frac{z_{k+1}(t)}{2\varepsilon}\times$$

$$\left(\frac{\partial\alpha_k}{\partial x_i}\right)^2(a_i^2 + b_i^2) + \Delta_{k+1}\left(1-\frac{1}{2m}\right)\left(\frac{z_{k+1}\Delta_{k+1}}{2m\beta}\right)^{\frac{1}{2m-1}} + K_{k+1}z_{k+1}(t), \tag{12}$$

则可得:

$$\dot{V}_{k+1} + y^{2m}(t) - (k+1)\beta\|\omega\|^{2m} \leqslant z_{k+1}(t)z_{k+2}(t) - z_1(t)g(z_1(t)) - \sum_{i=1}^{k+1}K_i z_i^2(t) +$$

$$\frac{(k+1)(k+2)}{4}\varepsilon y^2(t)\left(1+\frac{1}{1-\eta}\right),$$

故递归成立.

第 n 步:反复利用递归证明可构造出形如(2)的全局坐标变换和正定 Lyapunov函数以及光滑状态反馈.

$$V_n = V_{n-1} + \frac{1}{2}z_n^2(t) + \frac{n\varepsilon}{2(1-\eta)}\int_{t-d(t)}^{t} y^2(s)\,\mathrm{d}s, \tag{13}$$

$$u = -\left\{z_{n-1}(t) + \frac{z_n(t)}{2\varepsilon}(a_n^2 + b_n^2) + \sum_{i=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_i}(z_{i+1}(t) - \alpha_i) + \right.$$

$$\left.\sum_{i=1}^{n-1}\frac{z_n(t)}{2\varepsilon}\left(\frac{\partial\alpha_{n-1}}{\partial x_i}\right)^2(a_i^2 + b_i^2) + \Delta_n\left(1-\frac{1}{2m}\right)\left(\frac{z_n\Delta_n}{2m\beta}\right)^{\frac{1}{2m-1}} + K_n z_n(t)\right\}, \tag{14}$$

取

$$g(z_1(t)) = \frac{n(n+1)}{4}\varepsilon\tau_1^2(x_1(t))z_1(t)\left(1+\frac{1}{1-\eta}\right), \tag{15}$$

故

$$\dot{V}_n + y^{2m}(t) - n\beta\|\omega\|^{2m} \leqslant -\sum_{i=1}^{n}K_i z_i^2(t). \tag{16}$$

令 $\gamma^2 = n\beta$,由 $V_n(0) = 0$以及 $V_n(t)$的正定性可得

$$\int_0^t |y(s)|^{2m}\,\mathrm{d}s \leqslant \gamma^2\int_0^t\|\omega\|^{2m}\,\mathrm{d}s, \quad \forall t \geqslant 0 \tag{17}$$

易知, 当 $\omega = 0$时, 系统在原点全局渐近稳定.

2 数值例子和系统仿真

考虑含有时滞的非线性系统

$$\begin{cases} \dot{x}_1(t) = x_2(t) + \frac{1}{2}y(t) + y\left(t - \frac{1}{3}\right) + x_1^2(t)\omega, \\ \dot{x}_2(t) = u(t) + y(t) + y\left(t - \frac{1}{3}\right) + x_2^2(t)\omega, \\ y(t) = x_1(t). \end{cases} \tag{18}$$

选择适当的参数, 利用上述设计方法可得控制器:

$$\begin{cases} \alpha_1 = x_1^5(t) + \frac{11}{2}x_1(t), \\ u = -\left\{x_1(t) + \frac{\partial \alpha_1}{\partial x_1}x_2(t) + (x_2(t) + \alpha_1) \times \left[3 + 2\left(\frac{\partial \alpha_1}{\partial x_1}\right)^2 + \left(5x_1^6 + \frac{11}{2}x_1^2 + x_2^2\right)^2\right]\right\}. \end{cases} \tag{19}$$

当 $\omega = 0$时, 取初始值 $x_1(0) = -1$, $x_2(0) = 2$, 所得仿真结果如下图所示.

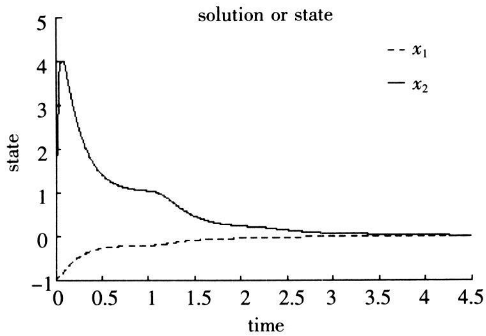

图 1 系统状态响应图

Fig.1 The response diagram of the system states

3 结 论

本文利用 Backstepping方法研究了一类非线性时滞系统的鲁棒 H_∞几乎干扰解耦. 通过技巧性地构造正定 Lyapunov函数, 解决了时滞项的问题, 本质改进了 Backstepping方法.

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