Quark Model Study of Heavy Baryons

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Abstract The heavy baryon spectra were studied in the framework of quark model. The three-body Schrodinger equation was solved numerically by employing the Gaussian Expansion Method. The results showed that the heavy baryons can be described well in the constituent quark models. The present work is limited to the heavy baryon with one heavy flavor quark.

Key words: heavy baryon spectra, Gaussian Expansion Method, three-body Schrodinger equation

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重重子的夸克模型研究

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[摘要] 在夸克模型的框架下研究了重重子谱, 通过高斯展开方法解薛定谔方程求得重重子的夸克模型研究. 结果充分说明了重重子可以用组分夸克模型来描述. 这篇文章的工作仅限于含一个重味夸克的重重子.

[关键词] 重重子谱, 高斯展开方法, 三体薛定谔方程

In the past years continuous progress has been made in the field of heavy baryons. The heavy baryons: charmed and bottomed baryons consist of a heavy quark (c and b) and two light quarks (double charmed baryons with two charm quarks and one light quark are also discovered). Most of the ground state heavy baryons have been found experimentally[1] and several new excited charmed baryon states have been observed recently by BaBar, Belle and CLEO collaborations[2]. The heavy baryons provide an excellent way of testing our understanding of quantum chromodynamics (QCD), the flavor independence of confinement force, the effect of one-gluon exchange and so on. It allows us to study how the interaction between quarks evolves from light to heavy system, and it is a good laboratory for testing the predictions of different theoretical approaches. There are many approaches or models used to study the heavy baryons. QCD sum rules method within the frame of heavy quark effective theory (HQET)[3,4] calculated the masses of p-wave excited heavy baryons, to the $1/m_Q$ order. The so called relativistic three-quark models[5] were used to study the properties of heavy baryons. Lattice QCD was also applied to study the spectroscopy of heavy baryons[6]. However, to our knowledge, only the constituent quark model is pursuing to describe hadron properties[7-9] and hadron-hadron interactions[10,11] simultaneously so far, and a lot of successes have been achieved. The heavy baryons should be described better than the light baryons in constituent quark models[12], because the appearance of heavy quark reduces the relativistic effect greatly. The present work is to study the heavy baryons in the framework of constituent quark model by using the Gaussian Expansion Method (GEM)[13], which differs from other work, to solve the three-body Schrodinger equation.

1 Constituent Quark Model

The model used has been successfully applied to describe hadron properties and baryon-baryon interactions

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The detail of the model can be found in Ref [7, 10]. Here only the Hamiltonian and the construction of wavefunctions are given. The Hamiltonian is

$$H = \sum_{i=1}^{3}\left(m_i + \frac{p_i^2}{2m_i}\right) - T_G + \sum_{j>i=1}^{3}\left(V_{ij}^{\mathrm{conf}}(\boldsymbol{r}_{ij}) + V_{ij}^{\mathrm{OGE}}(\boldsymbol{r}_{ij})\right), \tag{1}$$

$$V_{ij}^{\mathrm{conf}}(\boldsymbol{r}_{ij}) = (\lambda_i^c \cdot \lambda_j^c)(-\alpha_c r_{ij}^2 - \Delta), \tag{2}$$

$$V_{ij}^{\mathrm{OGE}}(\boldsymbol{r}_{ij}) = \frac{1}{4}\alpha_s \lambda_i^c \cdot \lambda_j^c\left[\frac{1}{r_{ij}} - \frac{\pi}{2}\delta(\boldsymbol{r}_{ij})\left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4\sigma_i \cdot \sigma_j}{3m_i m_j}\right)\right], \tag{3}$$

where all the symbols have their usual meaning. To solve the three-body Schrödinger equation, the delta function in the one-gluon exchange potential must be regularized[9]

$$\delta(\boldsymbol{r}_{ij}) = (\beta^2\pi)^{-\frac{3}{2}} e^{-\frac{r_{ij}^2}{\beta^2}}. \tag{4}$$

To take into account of the scale-dependence of strong coupling constant, an effective coupling constant is written by[9]

$$\alpha_s = \frac{\alpha_{s0}}{\ln\left[\frac{\mu^2 + \mu_0^2}{\Lambda_0^2}\right]}, \tag{5}$$

where μ is the reduced mass of the interacting quark-pair. The construction of wavefunctions are given below. There are two different cases for the studied heavy baryons: (qsQ) and (qqQ), where q denotes u or d quark, s quark for Ω_Q^-, and Q means b or c quark. So Ξ_Q and Ξ_Q^* belong to the former case (qsQ), and Λ_Q, Σ_Q, Σ_Q^* and Ω_Q^- belong to the latter case (qqQ).

(1) Wave functions of Ξ_Q and Ξ_Q^*: In this case, we consider three sets of rearrangement Jacobin coordinates (see Fig. 1) and refer them as a-type, b-type and c-type.

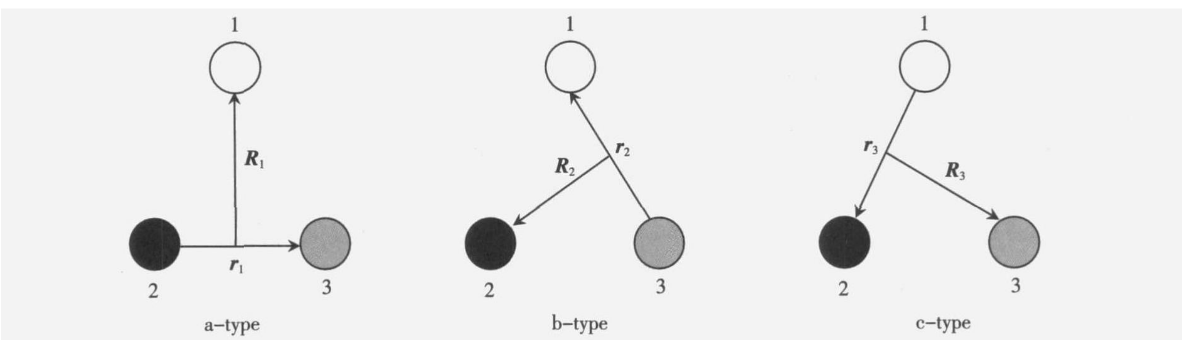

Fig. 1 Rearrangement Jacobian coordinates for Ξ_Q and Ξ_Q^*

The three quarks are all different, make the defining

$$\boldsymbol{r}_k = \boldsymbol{x}_i - \boldsymbol{x}_j, \quad \boldsymbol{R}_k = \boldsymbol{x}_k - (\boldsymbol{x}_i + \boldsymbol{x}_j)/2, \quad i, j, k = 1, 2, 3 \text{ (cyclicing)}, \tag{6}$$

we can construct three-body basis functions for the color-singlet, total spin and its z-component J, M and total isospin and its z-component T, T_z concretely as the following

$$\Phi_{JMTT_z\xi}^{(a)} = [[[\chi_{\frac{1}{2}}(2)\chi_{\frac{1}{2}}(3)]_s\chi_{\frac{1}{2}}(1)]_S[\phi_l(\boldsymbol{r}_1)\psi_L(\boldsymbol{R}_1)]_\mathcal{L}]_{JM}[[\eta_\tau(2)\eta_\tau(3)]_t\eta_\tau(1)]_{TT_Z}\Phi(c), \tag{7}$$

$$\Phi_{JMTT_z\xi}^{(b)} = [[[\chi_{\frac{1}{2}}(1)\chi_{\frac{1}{2}}(3)]_s\chi_{\frac{1}{2}}(2)]_S[\phi_l(\boldsymbol{r}_2)\psi_L(\boldsymbol{R}_2)]_\mathcal{L}]_{JM}[[\eta_\tau(1)\eta_\tau(3)]_t\eta_\tau(2)]_{TT_Z}\Phi(c), \tag{8}$$

$$\Phi_{JMTT_z\xi}^{(c)} = [[[\chi_{\frac{1}{2}}(1)\chi_{\frac{1}{2}}(2)]_s\chi_{\frac{1}{2}}(3)]_S[\phi_l(\boldsymbol{r}_3)\psi_L(\boldsymbol{R}_3)]_\mathcal{L}]_{JM}[[\eta_\tau(1)\eta_\tau(2)]_t\eta_\tau(3)]_{TT_Z}\Phi(c). \tag{9}$$

Where the quantum number set $\xi = \{s, S, l, L, \mathcal{L}, t\}$ are middle quantum numbers, $[\ ]$ means Clebsh-Gordan coupling, $\eta_{\tau\tau_z}(i)$, $\chi_{\frac{1}{2}m}(i)$ are the isospin and the spin function of the i-th quark, and ϕ_l and ψ_L are the two relative motion wavefunctions. The baryon wave function should be the superposition of these basis functions:

$$\Psi_{JMTT_z} = \sum_{\xi} a_\xi \Phi_{JMTT_z\xi}^{(a)} + \sum_{\xi'} b_{\xi'} \Phi_{JMTT_z\xi'}^{(b)} + \sum_{\xi''} c_{\xi''} \Phi_{JMTT_z\xi''}^{(c)}. \tag{10}$$

(2) Wave functions of Λ_Q, Σ_Q, Σ_Q^* and Ω_Q^-: In this case, there is a different quark which is denoted by

number '1', the white circle in the Fig 2

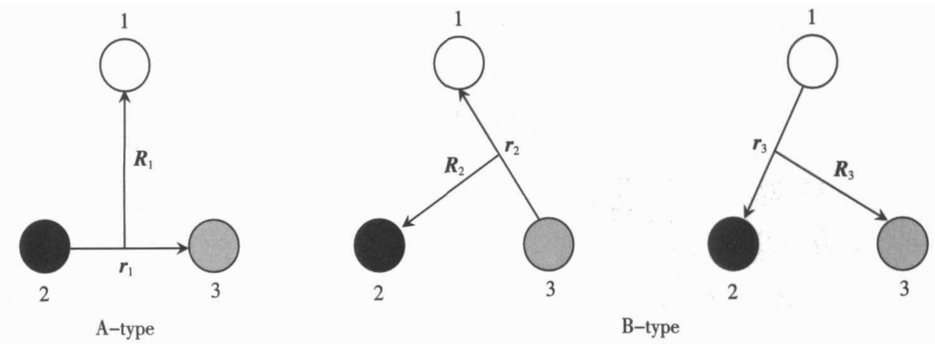

Fig. 2 Rearrangement Jacobian coordinates for Λ_Q, Σ_Q, Σ_Q^* and Ω_Q^-

The three basis functions are the same as before. Because there are two identical quarks, the total wave function should be antisymmetrized under the exchange of the two identical quarks. Defining

$$\Phi^{(A)}_{JMTT_z\xi} = \Phi^{(a)}_{JMTT_z\xi}, \qquad \Phi^{(B)}_{JMTT_z\xi} = \Phi^{(b)}_{JMTT_z\xi} + \Phi^{(c)}_{JMTT_z\xi}, \tag{11}$$

the baryon wavefunction can be written as

$$\Psi_{JMTT_z} = \sum_{\xi} A_\xi \Phi^{(A)}_{JMTT_z\xi} + \sum_{\xi'} B^{(B)}_{\xi'} \Phi_{JMTT_z\xi'}. \tag{12}$$

The mass of baryon, the two relative wavefunctions and the coefficients A_ξ and $B_{\xi'}$ (or the coefficients a_ξ, $b_{\xi'}$ and $c_{\xi''}$ in the first case) are to be determined by solving the Schrödinger equation

$$(H - E)\Psi_{JMTT_z} = 0 \tag{13}$$

with the Rayleigh–Litz variational principle.

2 Method of Calculation

To solve the above Schrödinger equation is very difficult. Here we employ the Gaussian expansion method (GEM), which developed by Hiyama et al[13], to solve the equation. In GEM, the relative motion wavefunction is expanded by a series of Gaussians with range parameters chosen to lie in a geometrical progression:

$$\phi_l(\boldsymbol{r}) = \sum_{n=1}^{n_{\max}} f_n \phi^{G}_{nl}(r) Y_{lm}(\hat{r}), \tag{14}$$

$$\phi^{G}_{nl}(r) = N_{nl} r^l e^{-\nu_n r^2}, \quad \nu_n = 1/r_n^2, \quad r_n = r_1 a^{n-1}, \quad (n = 1 - n_{\max}) \tag{15}$$

the normalization constant is

$$N_{nl} = \left[\frac{2^{l+2}(2\nu_n)^{l+\frac{3}{2}}}{\sqrt{\pi}(2l+1)!!}\right]^{\frac{1}{2}}. \tag{16}$$

f_n, $n = 1, \cdots, n_{\max}$ are variational parameters determined by the Eq (13), too $n_{\max}$, r_1, a (or $r_{\max} = r_1 a^{n_{\max}-1}$) are chosen to stabilized the results.

The calculation of matrix elements of color, spin and flavor part are simple and straight, which omitted here. The following is devoted to the evaluation of matrix elements of spatial part. For an arbitrary shape of $V(\boldsymbol{r}_c)$, its matrix elements can be written as follows:

$$\langle [\phi^{G}_{n_a l_a}(\boldsymbol{r}_a)\psi^{G}_{N_a L_a}(\boldsymbol{R}_a)]_{JM} | V(\boldsymbol{r}_c) | [\phi^{G}_{n_b l_b}(\boldsymbol{r}_b)\psi^{G}_{N_b L_b}(\boldsymbol{R}_b)]_{JM}\rangle, \tag{17}$$

where the ket- and bra-vectors are from different sets of Jacobian coordinators a-type and b-type. So we have to make coordinate transformations $(\boldsymbol{r}_a, \boldsymbol{R}_a)$, $(\boldsymbol{r}_b, \boldsymbol{R}_b) \to (\boldsymbol{r}_c, \boldsymbol{R}_c)$

$$\boldsymbol{r}_a = \alpha_{ac}\boldsymbol{r}_c + \beta_{ac}\boldsymbol{R}_c, \quad \boldsymbol{R}_a = \gamma_{ac}\boldsymbol{r}_c + \delta_{ac}\boldsymbol{R}_c, \quad \boldsymbol{r}_b = \alpha_{bc}\boldsymbol{r}_c + \beta_{bc}\boldsymbol{R}_c, \quad \boldsymbol{R}_b = \gamma_{bc}\boldsymbol{r}_c + \delta_{bc}\boldsymbol{R}_c, \tag{18}$$

and using the formula

$$r_a Y_{lm}(\boldsymbol{r}_a) = \sum_{\lambda=0}^{l}\left[\frac{4\pi(2l+1)!}{(2\lambda+1)!(2(l-\lambda)+1)!}\right]^{\frac{1}{2}}(\alpha_{ac}r_c)^{l-\lambda}(\beta_{ac}R_c)^{\lambda}[Y_\lambda(\boldsymbol{r}_c)Y_{l-\lambda}(\boldsymbol{R}_c)]_{lm},$$

the ket-vector:

$$[\phi^{G}_{n_a l_a}(\boldsymbol{r}_a)\Psi^{G}_{N_a L_a}(\boldsymbol{R}_a)]_{JM} = N_{n_a l_a} N_{N_a L_a} r_a^{\,l_a} R_a^{\,L_a} [Y_{l_a m_a}(\boldsymbol{r}_a) Y_{L_a M_a}(\boldsymbol{R}_a)]_{JM}\, e^{-\upsilon_a r_a^2 - \lambda_a^2 R_a^2}$$

can be recasted into function of $\boldsymbol{r}_c$ and $\boldsymbol{R}_c$

$$r_a^{\,l_a} R_a^{\,L_a} [Y_{l_a m_a}(\boldsymbol{r}_a) Y_{L_a M_a}(\boldsymbol{R}_a)]_{JM} =$$

$$(2l_a+1)(2L_a+1)\sum_{\lambda=0}^{l_a}\sum_{\Lambda=0}^{L_a}[C^{2\lambda}_{2l_a} C^{2\Lambda}_{2L_a}]^{\frac{1}{2}} \alpha_{ac}^{\,l_a-\lambda}\beta_{ac}^{\,\lambda}\gamma_{ac}^{\,L_a-\Lambda}\delta_{ac}^{\,\Lambda} r_c^{\,l_a+L_a-(\lambda+\Lambda)} R_c^{\,\lambda+\Lambda} \times$$

$$\sum_{l_c L_c}\begin{Bmatrix} l_a-\lambda & L_a-\Lambda & l_c \\ \lambda & \Lambda & L_c \\ l_a & L_a & J \end{Bmatrix} \langle l_a-\lambda\, L_a-\Lambda\, 0\, 0 \mid l_c\, 0\rangle \langle \lambda\, \Lambda\, 0\, 0 \mid L_c\, 0\rangle [Y_{l_c}(\boldsymbol{r}_c) Y_{L_c}(\boldsymbol{R}_c)]_{JM}.$$

So the former expression can be summarized in the form (denoting $\lambda+\Lambda$ as K)

$$[\phi^{G}_{n_a l_a}(\boldsymbol{r}_a)\Psi^{G}_{N_a L_a}(\boldsymbol{R}_a)]_{JM} = N_{n_a l_a} N_{N_a L_a}\sum_{l_c L_c}\sum_{K=0}^{l_a+L_a}\langle l_a L_a J \mid l_c L_c K\, J\rangle_{a\to c}\, r_c^{\,l_a+L_a-K} R_c^{\,K} \times$$

$$[Y_{l_c}(\boldsymbol{r}_c) Y_{L_c}(\boldsymbol{R}_c)]_{JM}\, e^{-\upsilon_a r_a^2 - \lambda_a^2 R_a^2},$$

where the expression

$$\langle l_a L_a J \mid l_c L_c K\, J\rangle_{a\to c} = (2l_a+1)(2L_a+1)[C^{2\lambda}_{2l_a} C^{2\Lambda}_{2L_a}]^{\frac{1}{2}} \alpha_{ac}^{\,l_a-\lambda}\beta_{ac}^{\,\lambda}\gamma_{ac}^{\,L_a-\Lambda}\delta_{ac}^{\,\Lambda} \times$$

$$\begin{Bmatrix} l_a-\lambda & L_a-\Lambda & l_c \\ \lambda & \Lambda & L_c \\ l_a & L_a & J \end{Bmatrix} \langle l_a-\lambda\, L_a-\Lambda\, 0\, 0 \mid l_c\, 0\rangle \langle \lambda\, \Lambda\, 0\, 0 \mid L_c\, 0\rangle.$$

We have similar expression for the bra-vectors in b-type.

The transformation coefficients $\langle l_a L_a J \mid l_c L_c K\, J\rangle_{a\to c}$ and $\langle l_b L_b J \mid l_c L_c K\, J\rangle_{b\to c}$ can be calculated and stored prior to the computation. Introduce the modified Bessel function $I_l(z) = (-i)^l j_l(iz)$, then

$$e^{-\upsilon_a r_a^2 - \lambda_a^2 R_a^2} e^{-\upsilon_b r_b^2 - \lambda_b^2 R_b^2} = e^{-\eta r_c^2 - 2\xi \boldsymbol{r}_c\cdot\boldsymbol{R}_c - \zeta R_c^2}\sum_{l=0}^{\infty} 4\pi\sqrt{2l+1}\, I_l(2\xi r_c R_c)[Y_l(\boldsymbol{r}_c) Y_l(\boldsymbol{R}_c)]_0. \tag{19}$$

Combining all these equations, and perform the integration over r_c and R_c, then the matrix elements can be worked out. The calculation of the overlap and the matrix elements of kinetic-energy operators is simpler than the above, only the coordinate transformation $(\boldsymbol{r}_a, \boldsymbol{R}_a) \to (\boldsymbol{r}_b, \boldsymbol{R}_b)$ is needed.

3 Result and Discussion

The parameters in Hamiltonian are determined by fitting the experimental masses of light and heavy baryons and are listed in Table 1. The calculated masses of heavy baryons for ground states and excited states are given in Table 2 and Table 3 respectively, compared with other theoretical predictions and the experimental values. The experimental data are taken from PDG[1].

Table 1 The parameters of the constituent quark model

Quark masses	$m_u = m_d = 313$ MeV	$m_s = 501$ MeV
	$m_c = 1\,729$ MeV	$m_b = 5\,073$ MeV
Confinement	$a_c = 27.75$ MeV fm^{-2}	
OGE	$\alpha_{s0} = 4.965$	$\Lambda_0 = 0.113$ fm^{-1}
	$\mu_0 = 36.976$ MeV	$\beta = 0.206$

From Table 2, one can see that our calculations are in good agreement with experiments, similar to the estimation of ref [14]. The good agreement also means that the color confinement is indeed flavor independent, because the same confining interaction is used for light baryons and heavy baryons. Coming to the excited states (negative-parity states), our theoretical results always are lower than the experimental values, although a reasonable agreement is obtained. We should keep in mind that there is a problem is still preliminary. Perhaps some inevitable deficiency with the model to calculate it or other reasons.

Table 2 Masses spectra of the ground sates of heavy baryons (in MeV).

Baryon	Quark content	$T(J^P)$	Our work	[Ebert]	Exp
Λ_c	udc	$0\left(\frac{1}{2}^+\right)$	2285	2297	2284.9(6)
Σ_c	uuc	$1\left(\frac{1}{2}^+\right)$	2471	2439	2451.3(7)
Σ_c^*	uuc	$1\left(\frac{3}{2}^+\right)$	2512	2518	2515.9(2.4)
Ξ_c'	usc	$\frac{1}{2}\left(\frac{1}{2}^+\right)$	2531	2578	2574.1(3.3)
Ξ_c^*	usc	$\frac{1}{2}\left(\frac{3}{2}^+\right)$	2630	2654	2647.4(2.0)
Ω_c	ssc	$0\left(\frac{1}{2}^+\right)$	2698	2698	2697.5(2.6)
Ω_c^*	ssc	$0\left(\frac{3}{2}^+\right)$	2743	2768	—
Λ_b	udb	$0\left(\frac{1}{2}^+\right)$	5610	5622	5624(9)
Σ_b	uub	$1\left(\frac{1}{2}^+\right)$	5816	5805	5807.8 ± 2.0 ± 1.7(CDF)
Σ_b^*	uub	$1\left(\frac{3}{2}^+\right)$	5830	5834	5829 ± 1.8 ± 1.8(CDF)
Ξ_b'	usb	$\frac{1}{2}\left(\frac{1}{2}^+\right)$	5858	5937	—
Ξ_b^*	usb	$\frac{1}{2}\left(\frac{3}{2}^+\right)$	5943	5963	—
Ω_b	ssb	$0\left(\frac{1}{2}^+\right)$	6034	6065	—
Ω_b^*	ssb	$0\left(\frac{3}{2}^+\right)$	6051	6088	—

The masses of the ground state and excited state heavy baryons consisting of two light (u, d, s) quarks and one heavy (c, b) quark are calculated in this work. For finite experiment data of excited state, here we only outline some results. We will do some further work in this field in the future.

Table 3 Masses spectra of the excited sates of heavy baryons (in MeV).

Baryon	Quark content	$T(J^P)$	Our work	Exp.
Λ_c	udc	$0\left(\frac{1}{2}^-\right)$	2505	2595.4
Λ_c	udc	$0\left(\frac{3}{2}^-\right)$	2505	2628.1
Λ_c	udc	$0\left(\frac{5}{2}^-\right)$	2783	—
Σ_c	uuc	$1\left(\frac{1}{2}^-\right)$	2695	2766
Σ_c	uuc	$1\left(\frac{3}{2}^-\right)$	2695	—
Σ_c	uuc	$1\left(\frac{5}{2}^-\right)$	2715	—
Ξ_c	usc	$\frac{1}{2}\left(\frac{1}{2}^-\right)$	2730	2789.2
Ξ_c	usc	$\frac{1}{2}\left(\frac{3}{2}^-\right)$	2730	2816.5
Ξ_c	usc	$\frac{1}{2}\left(\frac{5}{2}^-\right)$	2830	—
Ω_c	ssc	$0\left(\frac{1}{2}^-\right)$	2927	—
Ω_c	ssc	$0\left(\frac{3}{2}^-\right)$	2927	—
Ω_c	ssc	$0\left(\frac{5}{2}^-\right)$	2946	—

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