关于双二次域 $Q(\sqrt{m}, \sqrt{n})$ 的正规整基

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[摘要] 给出了等价的定义, 在存在正规整基的前提下证明了在等价的条件下双二次域的正规整基生成元惟一, 同时计算出了具体的生成元, 最后, 用具体例子验证了不同情形下的生成元惟一.

[关键词] 正规整基, 双二次域, 生成元
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On Normal Integral Basis of Biquadratic Number Fields $Q(\sqrt{m}, \sqrt{n})$

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Abstract: Equivalent definition is given first, then on the premise of the existence of NIB we prove the generator of NIB in Biquadratic number fields $Q(\sqrt{m}, \sqrt{n})$ is unique up to equivalence. At the same time, we obtain the generator of NIB. Finally, specific examples of different cases are given to verify the generator.

Key words: normal integral basis, biquadratic number fields, generator

设 K/Q 是有限伽罗瓦扩张, $G = \mathrm{Gal}(K/Q)$ 是伽罗瓦群, O_K 是 K 的代数整数环, 如果一个集合 $\{\ \ \}_G$ 满足

$$O_K = \ \ _G Z \ \ = [\ \ ,\ \ \ G],$$

则此集合被称为 O_K 的一组正规整基 (简写成 NIB), 其中 被称为正规整基的一个生成元. 如果 是正规整基的一个生成元, 则容易证明 $-$ 和 也是正规整基的生成元, 其中 G.

假设 和 是 O_K 的正规整基的两个生成元, 如果存在 G 满足 $=$, 即 和 是共轭或相差正负号, 那么我们称 和 是等价的, 记做 $\sim$. 我们不禁会问: 当 O_K 存在正规整基时, 正规整基的生成元在等价的意义下是否惟一呢? 本文求出了具有正规整基的双二次域 $Q(\sqrt{m}, \sqrt{n})$ 的正规整基的生成元的代表元, 即定理 2.

定理 1[1] 设 $K = Q(\sqrt{m}, \sqrt{n})$, 其中 m 和 n 是两个不同的无平方因子整数, 并且均不为 1, 记 $k = \frac{mn}{(m, n)^2}$, 那么:

(1) 令 $M = Q(\sqrt{m})$, 则 K 中的元素 是代数整数, 当且仅当 $N_{K/M}(\ \ )$ 和 $Tr_{K/M}(\ \ )$ 均是代数整数.

(2) 如果 $m \ \ 3 \pmod 4$, $n \ \ k \ \ 2 \pmod 4$, 则 $\left\{1, \sqrt{m}, \sqrt{n}, \frac{1}{2}(\sqrt{n}+\sqrt{k})\right\}$ 是 O_K 的一组整基, 并且 $D_K = 64mnk$.

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(3) 如果 $m \equiv 1 \pmod 4$, $n \equiv k \equiv 2$ 或 $3 \pmod 4$, 则 $\left\{1, \frac{1+\sqrt{m}}{2}, \sqrt{n}, \frac{\sqrt{n}+\sqrt{k}}{2}\right\}$ 是 O_K 的一组整基, 并且 $D_K = 16mnk$.

(4) 如果 $m \equiv n \equiv k \equiv 1 \pmod 4$, 则 $\left\{1, \frac{1}{4}(1+\sqrt{m})(1+\sqrt{k}), \frac{1}{2}(1+\sqrt{m}), \frac{1}{2}(1+\sqrt{n})\right\}$ 是 O_K 的一组整基, $D_K = mnk$.

引理 1[2,3] 记 $K = \boldsymbol{Q}(\sqrt{m}, \sqrt{n})$, m, n 都是无平方因子的整数. 则 $K/\boldsymbol{Q}$ 具有正规整基当且仅当 $m \equiv n \equiv 1 \pmod 4$.

1 定理的证明

本文的主要目的是证明双二次域 $\boldsymbol{Q}(\sqrt{m}, \sqrt{n})$ 在存在正规整基时, 正规整基是惟一的, 为了证明这一结论, 我们先来做些准备工作:

记 $K = \boldsymbol{Q}(\sqrt{m}, \sqrt{n})$, m, n 都是无平方因子且不相等的整数, $m \equiv n \equiv 1 \pmod 4$, $k = \frac{mn}{(m, n)^2} \equiv 1 \pmod 4$, $\mathrm{disc}(K) = mnk$. 注意 $\boldsymbol{Q}(\sqrt{m}, \sqrt{n}) = \boldsymbol{Q}(\sqrt{m}, \sqrt{k}) = \boldsymbol{Q}(\sqrt{n}, \sqrt{k}) = \boldsymbol{Q}(\sqrt{m}, \sqrt{n}, \sqrt{k})$. $\mathrm{Gal}(K/\boldsymbol{Q}) = \{id, \sigma_1, \sigma_2, \sigma_3\}$.

由 $m, n, k = \frac{mn}{(m, n)^2}$ 的正负号知 m, n, k 中必有一个为正, 再由 m, n, k 三者的对称性, 不妨假设 $m > 0$,

$$O_K = Z + Z\frac{(1+\sqrt{m})(1+\sqrt{k})}{4} + Z\frac{1+\sqrt{m}}{2} + Z\frac{1+\sqrt{n}}{2}.$$

引理 2 在四次域 $\boldsymbol{Q}(\sqrt{m}, \sqrt{n})$ 中, 当 $m > 0$ 且 $m \equiv n \equiv 1 \pmod 4$ 时,

$$\frac{1+\sqrt{k}}{2} = \frac{1+m_0}{2} + 2\frac{1+\sqrt{m}}{2}\frac{1+\sqrt{k}}{2} - \frac{1+\sqrt{m}}{2} - m_0\frac{1+\sqrt{n}}{2},$$

其中 $k = \frac{mn}{(m, n)^2}$, $m_0 = \frac{m}{(m, n)}$.

下面我们来证明这一结论:

定理 2 设 $K = \boldsymbol{Q}(\sqrt{m}, \sqrt{n})$ (其中 m, n 是两个不同的无平方因子整数, 并且均不为 1), $m \equiv n \equiv 1 \pmod 4$, 那么, $K/\boldsymbol{Q}$ 正规整基生成元在等价的条件下惟一.

证明 要完成该定理的证明必须分当 $m > 0$ 时, $m_0 = \frac{m}{(m, n)} \equiv 1 \pmod 4$ 和 $m_0 = \frac{m}{(m, n)} \equiv 3 \pmod 4$ 两种情况来讨论.

令 $\alpha = a + b\frac{1+\sqrt{m}}{2}\frac{1+\sqrt{k}}{2} + c\frac{1+\sqrt{m}}{2} + d\frac{1+\sqrt{n}}{2} \in O_K$, 其中 $a, b, c, d \in \mathbf{Z}$, 则

$$\sigma_1(\alpha) = a + b\frac{1+\sqrt{m}}{2}\frac{1-\sqrt{k}}{2} + c\frac{1+\sqrt{m}}{2} + d\frac{1-\sqrt{n}}{2} = a + b\frac{1+\sqrt{m}}{2}\left(1 - \frac{1+\sqrt{k}}{2}\right) + c\frac{1+\sqrt{m}}{2} + d\left(1 - \frac{1+\sqrt{n}}{2}\right) = (a+d) - b\frac{1+\sqrt{m}}{2}\frac{1+\sqrt{k}}{2} + (b+c)\frac{1+\sqrt{m}}{2} - d\frac{1+\sqrt{n}}{2},$$

$$\sigma_2(\alpha) = a + b\frac{1-\sqrt{m}}{2}\frac{1-\sqrt{k}}{2} + c\frac{1-\sqrt{m}}{2} + d\frac{1+\sqrt{n}}{2} = a + b\left(1 - \frac{1+\sqrt{m}}{2}\right)\left(1 - \frac{1+\sqrt{k}}{2}\right) + c\left(1 - \frac{1+\sqrt{m}}{2}\right) + d\frac{1+\sqrt{n}}{2} = \left(a + b\frac{1-m_0}{2} + c\right) - b\frac{1+\sqrt{m}}{2}\frac{1+\sqrt{k}}{2} - c\frac{1+\sqrt{m}}{2} + (bm_0 + d)\frac{1+\sqrt{n}}{2},$$

$$\sigma_3(\alpha) = a + b\frac{1-\sqrt{m}}{2}\frac{1+\sqrt{k}}{2} + c\frac{1-\sqrt{m}}{2} + d\frac{1-\sqrt{n}}{2} =$$

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$$a+b\left(1-\frac{1+\sqrt{m}}{2}\right)\frac{1+\sqrt{k}}{2}+c\left(1-\frac{1+\sqrt{m}}{2}\right)+d\left(1-\frac{1+\sqrt{n}}{2}\right)=$$

$$\left(a+b\frac{1+m_0}{2}+c+d\right)+b\frac{1+\sqrt{m}}{2}\frac{1+\sqrt{k}}{2}-(b+c)\frac{1+\sqrt{m}}{2}-(bm_0+d)\frac{1+\sqrt{n}}{2}.$$

那么要使　是正规整基生成元,则只需

$$\left(\begin{array}{c} \\ {}_1(\ ) \\ {}_2(\ ) \\ {}_3(\ ) \end{array}\right)=A\left(\begin{array}{c} 1 \\ \frac{1+\sqrt{m}}{2}\frac{1+\sqrt{k}}{2} \\ \frac{1+\sqrt{m}}{2} \\ \frac{1+\sqrt{n}}{2} \end{array}\right),$$

其中

$$A=\left(\begin{array}{cccc} a & b & c & d \\ a+b\frac{1-m_0}{2}+c & -b & -c & bm_0+d \\ a+d & -b & b+c & -d \\ a+b\frac{1+m_0}{2}+c+d & b & -(b+c) & -(bm_0+d) \end{array}\right)$$

满足 $\det A=\ 1$,而 $\det A=b(b+2c)(2d+bm_0)(4a+b+2c+2d)$,即 $b(b+2c)(2d+bm_0)(4a+b+2c+2d)=\ 1$.

从而

$$\begin{cases} b=\ 1, \\ b+2c=\ 1, \\ 2d+bm_0=\ 1, \\ 4a+b+2c+2d=\ 1. \end{cases} \tag{1}$$

情况 **1**　当 $m>0$, $m_0=\frac{m}{(m,n)}\ \ 1(\mathrm{mod}\ 4)$ 时,即 $m_0=4l+1$型,其中 $l\ \ \mathbf{Z}$,方程组(1)的整数解如下:

$$\begin{cases} a=\frac{m_0-1}{4}, \\ b=1, \\ c=0, \\ d=\frac{1-m_0}{2}; \end{cases} \quad \begin{cases} a=\frac{1-m_0}{4}, \\ b=-1, \\ c=0, \\ d=\frac{m_0-1}{2}; \end{cases} \quad \begin{cases} a=\frac{m_0-1}{4}, \\ b=1, \\ c=0, \\ d=-\frac{1+m_0}{2}; \end{cases} \quad \begin{cases} a=\frac{1-m_0}{4}, \\ b=-1, \\ c=0, \\ d=\frac{m_0+1}{2}; \end{cases}$$

$$\begin{cases} a=\frac{m_0-1}{4}, \\ b=1, \\ c=-1, \\ d=\frac{1-m_0}{2}; \end{cases} \quad \begin{cases} a=\frac{1-m_0}{4}, \\ b=-1, \\ c=1, \\ d=\frac{m_0-1}{2}; \end{cases} \quad \begin{cases} a=\frac{m_0+3}{4}, \\ b=1, \\ c=-1, \\ d=-\frac{1+m_0}{2}; \end{cases} \quad \begin{cases} a=-\frac{3+m_0}{4}, \\ b=-1, \\ c=1, \\ d=\frac{m_0+1}{2}. \end{cases}$$

$${}_1=\frac{m_0-1}{4}+\frac{1+\sqrt{m}}{2}\frac{1+\sqrt{k}}{2}+\frac{1-m_0}{2}\frac{1+\sqrt{n}}{2},$$

$${}_1=\frac{1-m_0}{4}-\frac{1+\sqrt{m}}{2}\frac{1+\sqrt{k}}{2}+\frac{m_0-1}{2}\frac{1+\sqrt{n}}{2},$$

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$${}_2 = \frac{m_0-1}{4} + \frac{1+\sqrt{m}}{2}\frac{1+\sqrt{k}}{2} - \frac{1+m_0}{2}\frac{1+\sqrt{n}}{2},$$

$${}_2 = \frac{1-m_0}{4} - \frac{1+\sqrt{m}}{2}\frac{1+\sqrt{k}}{2} + \frac{1+m_0}{2}\frac{1+\sqrt{n}}{2},$$

$${}_3 = \frac{m_0-1}{4} + \frac{1+\sqrt{m}}{2}\frac{1+\sqrt{k}}{2} - \frac{1+\sqrt{m}}{2} + \frac{1-m_0}{2}\frac{1+\sqrt{n}}{2},$$

$${}_3 = \frac{1-m_0}{4} - \frac{1+\sqrt{m}}{2}\frac{1+\sqrt{k}}{2} + \frac{1+\sqrt{m}}{2} + \frac{m_0-1}{2}\frac{1+\sqrt{n}}{2},$$

$${}_4 = \frac{m_0+3}{4} + \frac{1+\sqrt{m}}{2}\frac{1+\sqrt{k}}{2} - \frac{1+\sqrt{m}}{2} - \frac{1+m_0}{2}\frac{1+\sqrt{n}}{2},$$

$${}_4 = -\frac{m_0+3}{4} - \frac{1+\sqrt{m}}{2}\frac{1+\sqrt{k}}{2} + \frac{1+\sqrt{m}}{2} + \frac{1+m_0}{2}\frac{1+\sqrt{n}}{2}.$$

因为

$${}_1(\ {}_1) = \frac{m_0-1}{4} + \frac{1+\sqrt{m}}{2}\frac{1-\sqrt{k}}{2} + \frac{1-m_0}{2}\frac{1-\sqrt{n}}{2} =$$

$$\frac{m_0-1}{4} + \frac{1+\sqrt{m}}{2}\left(1-\frac{1+\sqrt{k}}{2}\right) + \frac{1-m_0}{2}\left(1-\frac{1+\sqrt{n}}{2}\right) = \ {}_3,$$

$${}_2(\ {}_1) = \frac{m_0-1}{4} + \frac{1-\sqrt{m}}{2}\frac{1-\sqrt{k}}{2} + \frac{1-m_0}{2}\frac{1+\sqrt{n}}{2} =$$

$$\frac{m_0-1}{4} + \left(1-\frac{1+\sqrt{m}}{2}\right)\left(1-\frac{1+\sqrt{k}}{2}\right) + \frac{1-m_0}{2}\frac{1+\sqrt{n}}{2} = \ {}_2,$$

$${}_3(\ {}_1) = \frac{m_0-1}{4} + \frac{1-\sqrt{m}}{2}\frac{1+\sqrt{k}}{2} + \frac{1-m_0}{2}\frac{1-\sqrt{n}}{2} =$$

$$\frac{m_0-1}{4} + \left(1-\frac{1+\sqrt{m}}{2}\right)\frac{1+\sqrt{k}}{2} + \frac{1-m_0}{2}\left(1-\frac{1+\sqrt{n}}{2}\right) = \ {}_4,$$

所以

$${}_1 = -\ {}_1,\quad {}_1\ {}_3 = -\ {}_3,\quad {}_1\ {}_2 = -\ {}_2,\quad {}_1\ {}_4 = -\ {}_4.$$

因此 $K = \boldsymbol{Q}(\sqrt{m},\sqrt{n})$, 当 $m > 0$, $m_0 = \dfrac{m}{(m,n)}\quad 1(\bmod 4)$ 时, 正规整基生成元在等价的条件下惟一.

情况 **2** 当 $m > 0$, $m_0 = \dfrac{m}{(m,n)}\quad 3(\bmod 4)$ 时, 即 $m_0 = 4l+3$ 型, 其中 $l\quad \boldsymbol{Z}$, 方程组(1)的整数解如下:

$$\begin{cases} a = \dfrac{m_0+1}{4}, \\ b = 1, \\ c = -1, \\ d = \dfrac{1-m_0}{2}; \end{cases} \quad \begin{cases} a = -\dfrac{m_0+1}{4}, \\ b = -1, \\ c = 1, \\ d = \dfrac{m_0-1}{2}; \end{cases} \quad \begin{cases} a = \dfrac{m_0+1}{4}, \\ b = 1, \\ c = -1, \\ d = -\dfrac{1+m_0}{2}; \end{cases} \quad \begin{cases} a = -\dfrac{m_0+1}{4}, \\ b = -1, \\ c = 1, \\ d = \dfrac{m_0+1}{2}; \end{cases}$$

$$\begin{cases} b = \dfrac{m_0+1}{4}, \\ b = 1, \\ c = 0, \\ d = -\dfrac{1+m_0}{2}; \end{cases} \quad \begin{cases} a = -\dfrac{m_0+1}{4}, \\ b = -1, \\ c = 0, \\ d = \dfrac{m_0+1}{2}; \end{cases} \quad \begin{cases} a = \dfrac{m_0-3}{4}, \\ b = 1, \\ c = 0, \\ d = \dfrac{1-m_0}{2}; \end{cases} \quad \begin{cases} a = \dfrac{3-m_0}{4}, \\ b = -1, \\ c = 0, \\ d = \dfrac{m_0-1}{2}. \end{cases}$$

$${}_1 = \frac{1+m_0}{4} + \frac{1+\sqrt{m}}{2}\frac{1+\sqrt{k}}{2} - \frac{1+m_0}{2}\frac{1+\sqrt{n}}{2},$$

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$$_1 = -\frac{1+m_0}{4} - \frac{1+\sqrt{m}}{2}\,\frac{1+\sqrt{k}}{2} + \frac{1+m_0}{2}\,\frac{1+\sqrt{n}}{2},$$

$$_2 = \frac{m_0-3}{4} + \frac{1+\sqrt{m}}{2}\,\frac{1+\sqrt{k}}{2} + \frac{1-m_0}{2}\,\frac{1+\sqrt{n}}{2},$$

$$_2 = \frac{3-m_0}{4} - \frac{1+\sqrt{m}}{2}\,\frac{1+\sqrt{k}}{2} - \frac{1-m_0}{2}\,\frac{1+\sqrt{n}}{2},$$

$$_3 = \frac{m_0+1}{4} + \frac{1+\sqrt{m}}{2}\,\frac{1+\sqrt{k}}{2} - \frac{1+\sqrt{m}}{2} - \frac{1+m_0}{2}\,\frac{1+\sqrt{n}}{2},$$

$$_3 = -\frac{1+m_0}{4} - \frac{1+\sqrt{m}}{2}\,\frac{1+\sqrt{k}}{2} + \frac{1+\sqrt{m}}{2} + \frac{1+m_0}{2}\,\frac{1+\sqrt{n}}{2},$$

$$_4 = \frac{1+m_0}{4} + \frac{1+\sqrt{m}}{2}\,\frac{1+\sqrt{k}}{2} - \frac{1+\sqrt{m}}{2} + \frac{1-m_0}{2}\,\frac{1+\sqrt{n}}{2},$$

$$_4 = -\frac{1+m_0}{4} - \frac{1+\sqrt{m}}{2}\,\frac{1+\sqrt{k}}{2} + \frac{1+\sqrt{m}}{2} - \frac{1-m_0}{2}\,\frac{1+\sqrt{n}}{2}.$$

同情况 1一样, 通过计算可得出 $_1(\ _1) = \ _3,\ _2(\ _1) = \ _2,\ _3(\ _1) = \ _4$. 所以

$$_1 = -\ _1,\ _1\ _3 = -\ _3,\ _1\ _2 = -\ _2,\ _1\ _4 = -\ _4.$$

因此 $K = \boldsymbol{Q}(\sqrt{m}, \sqrt{n})$, 当 $m > 0$, $m_0 = \frac{m}{(m, n)}\ \ 3(\mathrm{mod}\ 4)$ 时, 正规整基生成元在等价的条件下惟一.

综上所述: 已经完成了上述定理的证明.

2 例子

例 **1** $K = \boldsymbol{Q}(\sqrt{33}, \sqrt{21})$.

即 $m = 33$, $n = 21$, $k = \frac{mn}{(m, n)^2}\ \ 1(\mathrm{mod}\ 4)$, $m_0 = \frac{m}{(m, n)}\ \ 3(\mathrm{mod}\ 4)$.

令 $\ = a + b\frac{1+\sqrt{33}}{2}\,\frac{1+\sqrt{77}}{2} + c\frac{1+\sqrt{33}}{2} + d\frac{1+\sqrt{21}}{2}$, 其中 $a, b, c, d\ \ \mathbf{Z}$

方程组(1)的整数解如下:

$$\begin{cases} a = 3, \\ b = 1, \\ c = 0, \\ d = -6, \end{cases} \quad \begin{cases} a = -3, \\ b = -1, \\ c = 0, \\ d = 6; \end{cases} \quad \begin{cases} a = 2, \\ b = 1, \\ c = 0, \\ d = -5, \end{cases} \quad \begin{cases} a = -2, \\ b = -1, \\ c = 0, \\ d = 5; \end{cases}$$

$$\begin{cases} a = 3, \\ b = 1, \\ c = -1, \\ d = -6, \end{cases} \quad \begin{cases} a = -3, \\ b = -1, \\ c = 1, \\ d = 6; \end{cases} \quad \begin{cases} a = 3, \\ b = 1, \\ c = -1, \\ d = -5, \end{cases} \quad \begin{cases} a = -3, \\ b = -1, \\ c = 1, \\ d = 5. \end{cases}$$

$$_1 = \frac{1+\sqrt{33}+\sqrt{77}-\sqrt{21}}{4}, \quad _1 = \frac{-1-\sqrt{33}-\sqrt{77}+\sqrt{21}}{4},$$

$$_2 = \frac{-1+\sqrt{33}+\sqrt{77}+\sqrt{21}}{4}, \quad _2 = \frac{1-\sqrt{33}-\sqrt{77}-\sqrt{21}}{4},$$

$$_3 = \frac{-1-\sqrt{33}+\sqrt{77}-\sqrt{21}}{4}, \quad _3 = \frac{1+\sqrt{33}-\sqrt{77}+\sqrt{21}}{4},$$

$$_4 = \frac{1-\sqrt{33}+\sqrt{77}+\sqrt{21}}{4}, \quad _4 = \frac{-1+\sqrt{33}-\sqrt{77}-\sqrt{21}}{4}.$$

通过计算可得到

$$_1(\ _1) = \ _3,\ _2(\ _1) = \ _2,\ _3(\ _1) = \ _4.$$

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所以有

$$_1 = -\ _1,\quad _1\ _3 = -\ _3,\quad _1\ _2 = -\ _2,\quad _1\ _4 = -\ _4.$$

因此 $K = Q(\sqrt{33}, \sqrt{77})$ 正规整基生成元在等价的条件下也是惟一的.

例 **2** $K = Q(\sqrt{5}, \sqrt{-35})$.

即 $m = 5, n = -35, k = \frac{mn}{(m, n)^2} = -7 \quad 1(\mathrm{mod}\ 4), m_0 = \frac{m}{(m, n)} = 1 \quad 1(\mathrm{mod}\ 4)$.

令 $= a + b\frac{1+\sqrt{5}}{2}\frac{1+\sqrt{-7}}{2} + c\frac{1+\sqrt{5}}{2} + d\frac{1+\sqrt{-35}}{2}$, 其中 $a, b, c, d \quad \mathbf{Z}$

满足方程组(1)的 共 8个如下:

$$_1 = \frac{1+\sqrt{5}+\sqrt{-7}+\sqrt{-35}}{4},\quad _1 = \frac{-1-\sqrt{5}-\sqrt{-7}-\sqrt{-35}}{4},$$

$$_2 = \frac{-1+\sqrt{5}+\sqrt{-7}-\sqrt{-35}}{4},\quad _2 = \frac{1-\sqrt{5}-\sqrt{-7}+\sqrt{-35}}{4},$$

$$_3 = \frac{-1-\sqrt{5}+\sqrt{-7}+\sqrt{-35}}{4},\quad _3 = \frac{1+\sqrt{5}-\sqrt{-7}-\sqrt{-35}}{4},$$

$$_4 = \frac{1-\sqrt{5}+\sqrt{-7}-\sqrt{-35}}{4},\quad _4 = \frac{-1+\sqrt{5}-\sqrt{-7}+\sqrt{-35}}{4}.$$

通过计算可得到

$$_1(\ _1) = \ _3,\quad _2(\ _1) = \ _2,\quad _3(\ _1) = \ _4.$$

所以有

$$_1 = -\ _1,\quad _1\ _3 = -\ _3,\quad _1\ _2 = -\ _2,\quad _1\ _4 = -\ _4.$$

因此 $K = Q(\sqrt{5}, \sqrt{-35})$ 正规整基生成元在等价的条件下也是惟一的.

例 **3** $K = Q(\sqrt{-3}, \sqrt{-7})$.

注意: $Q(\sqrt{-3}, \sqrt{-7}) = Q(\sqrt{21}, \sqrt{-3})$, 即可取 $m = 21, n = -3, k = \frac{mn}{(m, n)^2} \quad 1(\mathrm{mod}\ 4), m_0 = \frac{m}{(m, n)} \quad 3(\mathrm{mod}\ 4)$, 同例 1可得出

$$_1 = -\ _1,\quad _1\ _4 = -\ _4,\quad _1\ _2 = -\ _2,\quad _1\ _3 = -\ _3.$$

因此 $K = Q(\sqrt{-3}, \sqrt{-7})$ 正规整基生成元在等价的条件下也是惟一的.

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