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非线性不确定参数系统的鲁棒H_∞自适应控制

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[摘要] 研究了线性化不可控非线性参数系统的鲁棒H_∞干扰衰减问题, 推广了非线性系统模型. 利用加幂积分器和参数隔离技巧, 设计了使系统全局镇定的动态反馈律. 数值例子说明了结论的正确性.

[关键词] 加幂积分器, 鲁棒动态反馈, 虚拟控制器

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H_∞ Robust Adaptive Control for Uncertain Nonlinearly Parameterized Systems

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Abstract The robust H_∞ disturbance attenuation with internal stability for nonlinearly parameterized systems with uncontrollable linearization is discussed. The systematic model is developed. By adding a power integrator and using the parameter separation technique, an explicit robust dynamic feedback law that solves the problem of global stabilization is designed. Finally, illustrative example verifies the correctness of the conclusion.

Key words adding a power integrator, robust dynamic feedback, virtual controller

在非线性控制领域, 鲁棒H_∞控制和自适应控制是两个有意义且具有挑战性的问题. 过去几年里, 许多研究者对这些问题已经做了探讨, 并获得了一些重要结论[1-6]. 已存在的结果大多是关于鲁棒H_∞控制或自适应控制, 而两者结合的问题研究较少.

实际应用中经常会遇到需要克服环境干扰且满足系统自我镇定的控制问题, 因此探讨这两个问题的结合是非常必要的. 文献[7]提供了一种关于自适应控制的方法, 它的特点是自适应项具有一般化形式且系统不可控. 可是, 此方法用于输出反馈形式不成立[8]. 一旦添加状态输出项, 系统的稳定性不能保证. 利用[9]中的加幂积分器方法[10,11], 形成了一种建立连续自适应实用输出跟踪控制器的方法, 该控制器保证了闭环系统的所有状态是全局稳定的. 文献[12]研究了非线性鲁棒H_∞自适应控制问题, 一个精确的光滑鲁棒动态反馈律被设计, 系统形式也得到推广. 更有意义的是, 该文全面考虑了干扰问题与自适应问题的结合.

本文进一步泛化不可控非线性参数系统形式, 这种非线性参数动态系统的研究不仅具有理论价值而且具有重要的应用价值. 在非线性项上确界给定的前提下, 结合参数隔离技巧, 运用递归设计方法建立了一种光滑自适应控制器, 系统达到全局渐近稳定.

1 问题形成

本文研究了如下一类高阶非线性系统:

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$$
\begin{cases}
\dot{x}_1 = d_1(x, u, \theta, t)x_2^{p_1} + \phi_1(x_1, x_2, \theta) + g_1(x_1, x_2, \theta)\omega, \\
\dot{x}_2 = d_2(x, u, \theta, t)x_3^{p_2} + \phi_2(x_1, x_2, x_3, \theta) + g_2(x_1, x_2, x_3, \theta)\omega, \\
\quad \cdots \\
\dot{x}_n = d_n(x, u, \theta, t)u^{p_n} + \phi_n(x_1, \cdots, x_n, u, \theta) + g_n(x_1, \cdots, x_n, u, \theta)\omega, \\
y = x_1,
\end{cases} \tag{1}
$$

其中 $x \in \mathscr{R}^n$, $u \in \mathscr{R}$为系统状态和系统输入. p_i, $i = 1, 2, \cdots, n$是正奇数, $\theta \in \mathscr{R}^s$和 $\omega \in \mathscr{R}^q$分别是未知常向量和干扰信号. 映射 d_i: $\mathscr{R}^n \times \mathscr{R} \times \mathscr{R}^s \times \mathscr{R} \to \mathscr{R}$, ϕ_i: $\mathscr{R}^{i+1} \times \mathscr{R}^s \to \mathscr{R}$, g_i: $\mathscr{R}^{i+1} \times \mathscr{R}^s \to \mathscr{R}^{1\times q}$, $i = 1, 2, \cdots, n$假定为 C^1 函数且 $\phi_i(0, 0, \cdots, \theta) = 0$.

具有内稳定性的鲁棒H_∞ 自适应几乎干扰解耦问题(RAH$_\infty$ - ADD):

任给实数 $\gamma > 0$, 存在一个如下时变光滑鲁棒自适应动态反馈律

$$
\dot{\theta} = \chi(x, \theta, t), \quad u = u(x, \theta, t), \tag{2}
$$

且 $\chi(0, 0, t) = 0$, $u(0, 0, t) = 0$使得闭环系统(1)和(2)满足如下条件:

(1) 当 $\omega = 0$, 闭环系统(1)和(2)在平衡点全局渐近稳定;

(2) $\forall \omega \in L_{2m}$, $m = 1, 2, \cdots$, 闭环系统(1)和(2)从初始状态 $x(0) = 0$出发的解的响应满足:

$$
\int_0^t |y(s)|^{2mp_1} \mathrm{d}s \leqslant \gamma^2 \int_0^t \omega^{2n} \mathrm{d}s, \quad \forall t \geqslant 0
$$

令 $L_p = \left\{ z(t) \mid \int_0^t z(s)^p \mathrm{d}s < \infty, \quad p \geqslant 1 \right\}$. 为研究方便, 给出如下假设.

假设 1 $p_1 \geqslant p_2 \geqslant \cdots \geqslant p_n \geqslant 1$是正奇数.

假设 2 $\omega \leqslant \delta$, δ是一个实常数且 $\delta \geqslant 0$

假设 3 存在 C^∞ 函数 $\lambda_i(x_1, \cdots, x_i) > 0$, $\tau_i(x_1, \cdots, x_i) > 0$和 $\mu_i(x_1, \cdots, x_{i+1}, \theta) > 0$, 使得

$$
\lambda_i(x_1, \cdots, x_i) \leqslant d_i(x, u, \theta, t) \leqslant \mu_i(x_1, \cdots, x_{i+1}, \theta),
$$
$$
\tau_i(x_1, \cdots, x_i) \leqslant d_i(x, u, \theta, t) \leqslant \mu_i(x_1, \cdots, x_{i+1}, \theta).
$$

假设 4 对 $i = 1, 2, \cdots, n$, 存在 $\phi_{ij}(x_1, \cdots, x_i, \theta)$使得

$$
\phi_i(x_1, \cdots, x_{i+1}, \theta) = \sum_{j=0}^{p_i - 1} x_{i+1}^j \phi_{ij}(x_1, \cdots, x_i, \theta),
$$
$$
|\phi_{i,j}(x_1, \cdots, x_i, \theta)| \leqslant (|x_1|^{p_i - j} + \cdots + |x_i|^{p_i - j}) b_{ij}(x_1, \cdots, x_i, \theta),
$$

其中 $b_{i,j}(x_1, \cdots, x_i, \theta) \geqslant 0$, $j = 0, 1, \cdots, p_i - 1$是连续函数.

2 鲁棒自适应控制器

在证明定理 1之前, 首先介绍几个有用的引理.

引理 1 对任意正整数 m, n和任意实值函数 $\pi(x, y) > 0$,

$$
|x|^m |y|^n \leqslant \frac{m}{m+n} \pi(x, y) |x|^{m+n} + \frac{n}{m+n} \pi^{-\frac{m}{n}}(x, y) |y|^{m+n}.
$$

引理 2 对满足假设 2和 3的不确定非线性函数 $d_i(\cdot)$和 $\phi_i(\cdot)$, 存在一个常数 $\Theta \geqslant 1$和 C^∞ 函数 $\gamma_i(x_1, \cdots, x_{i+1}) \geqslant 0$, $\gamma_i(x_1, \cdots, x_i) \geqslant 0$, $\beta_i(x_1, \cdots, x_i) \geqslant 0$使得

$$
\mu_i(x_1, \cdots, x_{i+1}, \theta) \leqslant \gamma_i(x_1, \cdots, x_{i+1})\Theta,
$$
$$
|\phi_i(x_1, \cdots, x_{i+1}, \theta)| \leqslant \frac{\lambda_i(x_1, \cdots, x_i)}{2} |x_{i+1}|^{p_i} + (|x_1|^{p_i} + \cdots + |x_i|^{p_i}) \gamma_i(x_1, \cdots, x_i)\Theta,
$$
$$
g_i(x_1, \cdots, x_{i+1}, \theta) \leqslant \frac{\tau_i(x_1, \cdots, x_i)}{2} |x_{i+1}|^{p_i} + (|x_1|^{p_i} + \cdots + |x_i|^{p_i}) \beta_i(x_1, \cdots, x_i)\Theta.
$$

此引理在文献[7]中已经被详细证明过.

下面定理是本部分的主要结论.

定理 1 在满足假设 1, 3和 4的前提下, 存在一个形如(2)式的鲁棒自适应动态控制器使问题 RAH$_\infty$ - ADD 得到解决.

证明　应用加幂积分器技巧,递归设计一个鲁棒自适应动态反馈律,使系统(1)的 RAH∞ − ADD问题得以解决.

第 **1**步　令 $\tilde{\Theta}(t)=\Theta-\hat{\Theta}(t)$,其中 $\Theta\geqslant 1$是未知常量,$\hat{\Theta}(t)$是 Θ的估计值. 选取 $V_1(x_1,\tilde{\Theta})=\frac{1}{2}x_1^2+\frac{1}{2}\tilde{\Theta}^2$,可以得到:

$$\dot{V}_1+y^{2mp_1}-\beta\omega^{2m}\leqslant d_1(\cdot)x_1x_2^{p_1}+x_1\left[\frac{\lambda_1(\cdot)}{2}|x_2|^{p_1}+|x_1|^{p_1}\gamma_1(\cdot)\Theta+\frac{\tau_1(\cdot)}{2}|x_2|^{p_1}\omega+\right.$$

$$\left.|x_1|^{p_1}\beta_1(\cdot)\Theta\omega\right]-\tilde{\Theta}\cdot\dot{\hat{\Theta}}+x_1^{2mp_1}-\beta\omega^{2m}\leqslant$$

$$d_1(\cdot)x_1x_2^{p_1}+\frac{\lambda_1(\cdot)}{2}x_1|x_2|^{p_1}+\frac{\tau_1(\cdot)\delta}{2}x_1|x_2|^{p_1}+|x_1|^{p_1+1}\gamma_1(\cdot)\sqrt{1+\hat{\Theta}^2}+x_1^{2mp_1}+$$

$$\left(1-\frac{1}{2m}\right)\left(\frac{1}{2m\beta}\right)^{\frac{1}{2m-1}}\left(|x_1|^{p_1+1}\beta_1(\cdot)\sqrt{1+\hat{\Theta}^2}\right)^{\frac{2m}{2m-1}}+\left[|x_1|^{p_1+1}\gamma_1(\cdot)+|x_1|^{p_1+1}\beta_1(\cdot)\delta-\dot{\hat{\Theta}}\right]\tilde{\Theta},\quad(3)$$

取定下面的虚拟控制器:

$$x_2^*=-x_1\left[\frac{2n+2\tilde{\gamma}_1(x_1,\hat{\Theta})+2\left(1-\frac{1}{2m}\right)\left(\frac{1}{2m\beta}\right)^{\frac{1}{2m-1}}\left(|x_1|^{p_1+1}\beta_1(\cdot)\sqrt{1+\hat{\Theta}^2}\right)^{\frac{2m}{2m-1}}}{\lambda_1(\cdot)+\tau_1(\cdot)\delta}\right]^{\frac{1}{p_1}},$$

其中 $\tilde{\gamma}_1(x_1,\hat{\Theta})=\gamma_1(\cdot)\sqrt{1+\hat{\Theta}^2}+x_1^{2mp_1-p_1-1}$. 因为 $x_1x_2^{*p_1}\leqslant 0$,易根据假设 2和 3得出:

$$\dot{V}_1+y^{2mp_1}-\beta\omega^{2m}\leqslant-nx_1^{p_1+1}+\left[\mu_1(\cdot)+\mu_1(\cdot)\delta+\frac{\lambda_1(\cdot)+\tau_1(\cdot)\delta}{2}\right]|x_1||x_2^{p_1}-x_2^{*p_1}|+$$

$$(\Psi_1(x_1)-\dot{\hat{\Theta}}(t))(\tilde{\Theta}(t)+\eta_1),\quad(4)$$

其中 $\Psi_1(x_1)=|x_1|^{p_1+1}\gamma_1(\cdot)+|x_1|^{p_1+1}\beta_1(\cdot)\delta$, $\eta_1=0$.

递归步　假设在第 k步,存在一系列虚拟控制器: $x_1^*,\dots,x_{k+1}^*$,定义如下

$$\xi_1=x_1-x_1^*,\ \xi_2=x_2-x_2^*,\ \dots,\ \xi_{k+1}=x_{k+1}-x_{k+1}^*.$$

$$x_1^*=0,\ x_2^*=-\xi_1\alpha_1(x_1,\hat{\Theta}),\ \dots,\ x_{k+1}^*=-\xi_k\alpha_k(x_1,\dots,x_k,\hat{\Theta}),\ i=1,2,\dots,n.$$

且 $\alpha_1(x_1,\hat{\Theta})>0,\dots,\alpha_k(x_1,\dots,x_k,\hat{\Theta})>0$光滑. 所以,(1)的子系统 $(x_1,\dots,x_k)$满足:

$$\dot{V}_k(\xi_1,\dots,\xi_k,\tilde{\Theta})+y^{2mp_1}-k\beta\omega^{2m}\leqslant-(n-k+1)(\xi_1^{p_1+1}+\dots+\xi_k^{p_1+1})+$$

$$\left[\mu_k(\cdot)+\mu_k(\cdot)\delta+\frac{\lambda_k(\cdot)+\tau_k(\cdot)\delta}{2}\right]|\xi_k^{p_1-p_k+1}||x_{k+1}^{p_k}-x_{k+1}^{*p_k}|+(\Psi_k(\cdot)-\dot{\hat{\Theta}})(\tilde{\Theta}+\eta_k(\cdot)),\quad(5)$$

其中

$$V_k(\xi_1,\dots,\xi_k,\tilde{\Theta})=\sum_{j=1}^{k}\frac{\xi_j^{p_1-p_j+2}}{p_1-p_j+2}+\frac{\tilde{\Theta}^2}{2}\quad(6)$$

是一个正定且正常的 Lyapunov函数, $0\leqslant\Psi_k(\cdot)\leqslant(\xi_1^{p_1+1}+\dots+\xi_k^{p_1+1})\sigma_k(\xi_1,\dots,\xi_k,\hat{\Theta})$,其中,$\sigma_k(\cdot)$为 C^∞函数且 $\sigma_k(\cdot)\geqslant 0$.

第 $\boldsymbol{k+1}$步　考虑 Lyapunov函数

$$V_{k+1}(\xi_1,\dots,\xi_{k+1},\tilde{\Theta})=V_k(\xi_1,\dots,\xi_k,\tilde{\Theta})+\frac{\xi_{k+1}^{p_1-p_{k+1}+2}}{p_1-p_{k+1}+2}\quad(7)$$

则有:

$$\dot{V}_{k+1}(\cdot)+y^{2mp_1}-(k+1)\beta\omega^{2m}\leqslant-(n-k+1)(\xi_1^{p_1+1}+\dots+\xi_k^{p_1+1})+$$

$$\left[\mu_k(\cdot)+\mu_k(\cdot)\delta+\frac{\lambda_k(\cdot)+\tau_k(\cdot)\delta}{2}\right]|\xi_k^{p_1-p_k+1}||x_{k+1}^{p_k}-x_{k+1}^{*p_k}|+(\Psi_k(\cdot)-\dot{\hat{\Theta}})(\tilde{\Theta}+\eta_k(\cdot))+$$

$$d_{k+1}(\cdot)\xi_{k+1}^{p_1-p_{k+1}+1}x_{k+2}^{p_{k+1}}+\xi_{k+1}^{p_1-p_{k+1}+1}\left[\phi_{k+1}(\cdot)-\sum_{j=1}^{k}\frac{\partial x_{k+1}^*}{\partial x_j}(d_j(\cdot)x_{j+1}^{p_j}+\phi_j)\right]+$$

$$\xi_{k+1}^{p_1-p_{k+1}+1}\left[g_{k+1}(\cdot)-\sum_{j=1}^{k}\frac{\partial x_{k+1}^{*}}{\partial x_j}\cdot g_j(\cdot)\right]\omega-\xi_{k+1}^{p_1-p_{k+1}+1}\frac{\partial x_{k+1}^{*}}{\partial\hat{\Theta}}\cdot\dot{\hat{\Theta}}-\beta\omega^{2m}. \tag{8}$$

利用引理 2, 可以证出:

$$\xi_{k+1}^{p_1-p_{k+1}+1}\left[\phi_{k+1}(\cdot)-\sum_{j=1}^{k}\frac{\partial x_{k+1}^{*}}{\partial x_j}(d_j(\cdot)x_{j+1}^{p_j}+\phi_j)\right]\leqslant\frac{\lambda_{k+1}(\cdot)}{2}\left|\xi_{k+1}^{p_1-p_{k+1}+1}x_{k+2}^{p_{k+1}}\right|+$$
$$\frac{\sum_{l=1}^{k}\xi_l^{p_1+1}}{6}+\xi_{k+1}^{p_1+1}\nu_{k+1}(\cdot)\sqrt{\hat{\Theta}^2+1}+\left[\frac{\sum_{l=1}^{k}\xi_l^{p_1+1}}{6(\hat{\Theta}^2+1)(\eta_k^2+1)}+\xi_{k+1}^{p_1+1}\nu_{k+1}(\cdot)\right]\Theta, \tag{9}$$

其中 $\nu_{k+1}(\xi_1, \dots, \xi_{k+1}, \hat{\Theta})$ 是一个光滑正值函数. 类似的, 下面的不等式成立:

$$\xi_{k+1}^{p_1-p_{k+1}+1}\left[g_{k+1}(\cdot)-\sum_{j=1}^{k}\frac{\partial x_{k+1}^{*}}{\partial x_j}\cdot g_j(\cdot)\right]\omega\leqslant$$
$$\left[\frac{\tau_{k+1}(\cdot)}{2}\left|\xi_{k+1}^{p_1-p_{k+1}+1}x_{k+2}^{p_{k+1}}\right|+\frac{\sum_{l=1}^{k}\xi_l^{p_1+1}}{6\delta}+\xi_{k+1}^{p_1+1}\bar{\nu}_{k+1}(\cdot)\sqrt{\hat{\Theta}^2+1}\right]\omega+$$
$$\left[\frac{\sum_{l=1}^{k}\xi_l^{p_1+1}}{6\delta(\hat{\Theta}^2+1)(\eta_k^2+1)}+\xi_{k+1}^{p_1+1}\bar{\nu}_{k+1}(\cdot)\right]\omega\Theta, \tag{10}$$

其中 $\bar{\nu}_{k+1}(\xi_1, \dots, \xi_{k+1}, \hat{\Theta})$ 是一个光滑正值函数. 进而, 易得:

$$\xi_{k+1}^{p_1-p_{k+1}+1}\left[g_{k+1}(\cdot)-\sum_{j=1}^{k}\frac{\partial x_{k+1}^{*}}{\partial x_j}\cdot g_j(\cdot)\right]\omega-\beta\omega^{2m}\leqslant\frac{\tau_{k+1}(\cdot)\delta}{2}\left|\xi_{k+1}^{p_1-p_{k+1}+1}x_{k+2}^{p_{k+1}}\right|+\frac{\sum_{l=1}^{k}\xi_l^{p_1+1}}{6}+$$
$$\left[\frac{\sum_{l=1}^{k}\xi_l^{p_1+1}}{6\delta(\hat{\Theta}^2+1)(\eta_k^2+1)}+\xi_{k+1}^{p_1+1}\bar{\nu}_{k+1}(\cdot)\right]\omega\Theta+\left(1-\frac{1}{2m}\right)\left(\frac{1}{2m\beta}\right)^{\frac{1}{2m-1}}\left[\xi_{k+1}^{p_1+1}\bar{\nu}_{k+1}(\cdot)\sqrt{\hat{\Theta}^2+1}\right]^{\frac{2m}{2m-1}}. \tag{11}$$

对一个光滑函数 $\tilde{\nu}_{k+1}(\xi_1, \dots, \xi_{k+1}, \hat{\Theta})\geqslant 0$, 有:

$$\left[\mu_k(\cdot)+\bar{\mu}_k(\cdot)\delta+\frac{\lambda_k(\cdot)+\tau_k(\cdot)\delta}{2}\right]\left|\xi_k^{p_1-p_k+1}\right|\left|x_{k+1}^{p_k}-x_{k+1}^{*\,p_k}\right|\leqslant$$
$$\left[(1+\delta)\gamma_k(\cdot)+\frac{\lambda_k(\cdot)+\tau_k(\cdot)\delta}{2}\right]\left|\xi_k^{p_1-p_k+1}\right|\left|x_{k+1}^{p_k}-x_{k+1}^{*\,p_k}\right|\Theta\leqslant$$
$$\frac{\sum_{l=1}^{k}\xi_l^{p_1+1}}{6}+\xi_{k+1}^{p_1+1}\tilde{\nu}_{k+1}(\cdot)\sqrt{\hat{\Theta}^2+1}+\left[\frac{\sum_{l=1}^{k}\xi_l^{p_1+1}}{6\delta(\hat{\Theta}^2+1)(\eta_k^2+1)}+\xi_{k+1}^{p_1+1}\tilde{\nu}_{k+1}(\cdot)\right]\Theta. \tag{12}$$

把 (9), (11) 和 (12) 代入 (8) 可得:

$$\dot{V}_{k+1}(\cdot)+y^{2mp_1}-(k+1)\beta\omega^{2m}\leqslant-\left(n-k+\frac{1}{2}\right)(\xi_1^{p_1+1}+\cdots+\xi_k^{p_1+1})+d_{k+1}(\cdot)\xi_{k+1}^{p_1-p_{k+1}+1}\times$$
$$x_{k+2}^{p_{k+1}}+\frac{\lambda_{k+1}(\cdot)+\tau_{k+1}(\cdot)\delta}{2}\left|\xi_{k+1}^{p_1-p_{k+1}+1}x_{k+2}^{p_{k+1}}\right|+\xi_{k+1}^{p_1+1}[\nu_{k+1}(\cdot)+\tilde{\nu}_{k+1}(\cdot)]\sqrt{\hat{\Theta}^2+1}+$$
$$\left[\frac{\sum_{l=1}^{k}\xi_l^{p_1+1}}{2(\hat{\Theta}^2+1)(\eta_k^2+1)}+\xi_{k+1}^{p_1+1}(\nu_{k+1}(\cdot)+\bar{\nu}_{k+1}(\cdot)\delta+\tilde{\nu}_{k+1}(\cdot))\right]\Theta-\xi_{k+1}^{p_1-p_{k+1}+1}\frac{\partial x_{k+1}^{*}}{\partial\hat{\Theta}}\dot{\hat{\Theta}}+$$
$$\left(1-\frac{1}{2m}\right)\left(\frac{1}{2m\beta}\right)^{\frac{1}{2m-1}}\left[\xi_{k+1}^{p_1+1}\bar{\nu}_{k+1}(\cdot)\sqrt{\hat{\Theta}^2+1}\right]^{\frac{2m}{2m-1}}+(\Psi_k(\cdot)-\dot{\hat{\Theta}})(\Theta+\eta_k(\cdot)). \tag{13}$$

令

$$\Psi_{k+1}(\xi_1, \dots, \xi_{k+1}, \hat{\Theta})=\Psi(\cdot)+\frac{\sum_{l=1}^{k}\xi_l^{p_1+1}}{2(\hat{\Theta}^2+1)(\eta_k^2+1)}+\xi_{k+1}^{p_1+1}(\nu_{k+1}(\cdot)+\bar{\nu}_{k+1}(\cdot)\delta+\tilde{\nu}_{k+1}(\cdot)),$$
$$\eta_{k+1}(\xi_1, \dots, \xi_{k+1}, \hat{\Theta})=\eta_k(\cdot)+\xi_{k+1}^{p_1-p_{k+1}+1}\frac{\partial x_{k+1}^{*}}{\partial\hat{\Theta}}, \tag{14}$$

则,式(13)变为:

$$\dot V_{k+1}(\cdot) + y^{2mp_1} - (k+1)\beta\omega^{2m} \leqslant -\left(n-k+\frac{1}{2}\right)(\xi_1^{p_1+1}+\cdots+\xi_k^{p_1+1}) + d_{k+1}(\cdot)\xi_{k+1}^{p_1-p_{k+1}+1}\times$$
$$x_{k+2}^{p_{k+1}} + \frac{\lambda_{k+1}(\cdot)+\tau_{k+1}(\cdot)\delta}{2}|\xi_{k+1}^{p_1-p_{k+1}+1}x_{k+2}^{p_{k+1}}| + \xi_{k+1}^{p_1+1}[\nu_{k+1}(\cdot)+\nu_{k+1}(\cdot)]\sqrt{\hat\Theta^2+1}+$$
$$(\Psi_{k+1}(\cdot)-\dot{\hat\Theta})(\Theta+\eta_{k+1}(\cdot)) - \left[\frac{\sum_{l=1}^{k}\xi_l^{p_1+1}}{2(\hat\Theta^2+1)(\eta_k^2+1)} + \xi_{k+1}^{p_1+1}(\nu_{k+1}(\cdot)+\nu_{k+1}(\cdot)\delta+\nu_{k+1}(\cdot))\right]\eta_k(\cdot) -$$
$$\xi_{k+1}^{p_1-p_{k+1}+1}\frac{\partial x_{k+1}^*}{\partial\hat\Theta}\cdot\Psi_{k+1} + \left(1-\frac{1}{2m}\right)\left(\frac{1}{2m\beta}\right)^{\frac{1}{2m-1}}[\xi_{k+1}^{p_1+1}\nu_{k+1}(\cdot)\sqrt{\hat\Theta^2+1}]^{\frac{2m}{2m-1}}. \tag{15}$$

根据引理2不难证出:

$$\left|\left[\frac{\sum_{l=1}^{k}\xi_l^{p_1+1}}{2(\hat\Theta^2+1)(\eta_k^2+1)} + \xi_{k+1}^{p_1+1}(\nu_{k+1}(\cdot)+\nu_{k+1}(\cdot)\delta+\nu_{k+1}(\cdot))\right]\eta_k(\cdot) + \xi_{k+1}^{p_1-p_{k+1}+1}\frac{\partial x_{k+1}^*}{\partial\hat\Theta}\cdot\Psi_{k+1}\right| \leqslant$$
$$\frac{\sum_{l=1}^{k}\xi_l^{p_1+1}}{2} + \xi_{k+1}^{p_1+1}(\nu_{k+1}(\cdot)+\nu_{k+1}(\cdot)\delta+\nu_{k+1}(\cdot))\sqrt{\eta_k^2+1} + \xi_{k+1}^{p_1+1}\sum_{j=1}^{k+1}\beta_{(k+1),j}(\cdot), \tag{16}$$

其中 $\beta_{k+1,j}(\xi_1,\dots,\xi_{k+1},\hat\Theta)\geqslant 0$, $1\leqslant j\leqslant k$, $\beta_{k+1,k+1}(\xi_1,\dots,\xi_{k+1},\hat\Theta)\geqslant \xi_{k+1}^{p_1-p_{k+1}+1}\times|\partial x_{k+1}^*/\partial\hat\Theta|$ $\alpha_{k+1}(\cdot)$都是光滑函数. (16)代入(15),得出:

$$\dot V_{k+1}(\xi_1,\dots,\xi_{k+1},\Theta) + y^{2mp_1} - (k+1)\beta\omega^{2m} \leqslant -(n-k)(\xi_1^{p_1+1}+\cdots+\xi_k^{p_1+1}) +$$
$$d_{k+1}(\cdot)\xi_{k+1}^{p_1-p_{k+1}+1}x_{k+2}^{p_{k+1}} + \frac{\lambda_{k+1}+\tau_{k+1}\delta}{2}|\xi_{k+1}^{p_1-p_{k+1}+1}x_{k+2}^{p_{k+1}}| +$$
$$\xi_{k+1}^{p_1+1}\left(\gamma_{k+1}(\cdot) + \left(1-\frac{1}{2m}\right)\left(\frac{1}{2m\beta}\right)^{\frac{1}{2m-1}}[\xi_{k+1}^{p_1+1}\nu_{k+1}(\cdot)\sqrt{\hat\Theta^2+1}]^{\frac{2m}{2m-1}}\right) +$$
$$(\Psi_{k+1}(\cdot)-\dot{\hat\Theta})(\Theta+\eta_{k+1}(\cdot)), \tag{17}$$

其中

$$\gamma_{k+1}(\xi_1,\dots,\xi_{k+1},\hat\Theta) = (\nu_{k+1}(\cdot)+\nu_{k+1}(\cdot)\delta+$$
$$\nu_{k+1}(\cdot))(\sqrt{\eta_k^2(\cdot)+1}+\sqrt{\hat\Theta^2+1}) + \sum_{j=1}^{k+1}\beta_{(k+1),j}(\cdot) \geqslant 0$$

是光滑函数. 令

$$x_{k+2}^* = -\xi_{k+1}\left[\frac{2(n-k)+2\gamma_{k+1}(\cdot)+2\nu_{k+1}(\cdot)\sqrt{\hat\Theta^2+1}\left(1-\frac{1}{2m}\right)\left(\frac{1}{2m\beta}\right)^{\frac{1}{2m-1}}(\xi_{k+1}^{p_1+1}\nu_{k+1}(\cdot)\sqrt{\hat\Theta^2+1})^{\frac{1}{2m-1}}}{\lambda_{k+1}(\cdot)+\tau_{k+1}(\cdot)\delta}\right]^{\frac{1}{p_{k+1}}} =$$
$$-\xi_{k+1}\alpha_{k+1}(x_1,\dots,x_{k+1},\hat\Theta),$$

且利用假设3有:

$$\dot V_{k+1}(\xi_1,\dots,\xi_{k+1},\Theta) + y^{2mp_1} - (k+1)\beta\omega^{2m} \leqslant -(n-k)(\xi_1^{p_1+1}+\cdots+\xi_{k+1}^{p_1+1}) +$$
$$\left(\mu_{k+1}(\cdot)+\mu_{k+1}(\cdot)\delta+\frac{\lambda_{k+1}(\cdot)+\tau_{k+1}(\cdot)\delta}{2}\right)|\xi_{k+1}^{p_1-p_{k+1}+1}||x_{k+2}^{p_{k+1}} - x_{k+2}^{*\,p_{k+1}}| +$$
$$(\Psi_{k+1}(\cdot)-\dot{\hat\Theta})(\Theta+\eta_{k+1}(\cdot)). \tag{18}$$

至此,递归步证明完成.

第 n 步　重复利用上面的步骤,可得到一个正定且正常的Lyapunov函数

$$V_n(\xi_1,\dots,\xi_n,\Theta) = V_{n-1}(\xi_1,\dots,\xi_{n-1},\Theta) + \frac{\xi_n^{p_1-p_n+2}}{p_1-p_n+2} \tag{19}$$

和一个动态反馈律 $u^* = x_{n+1}^*(\xi_1,\dots,\xi_n,\Theta) = -\xi_n\alpha_n(\cdot)$,使得:

$$\dot V_n(\xi_1,\dots,\xi_n,\Theta) + y^{2mp_1} - n\beta\omega^{2m} \leqslant -(\xi_1^{p_1+1}+\cdots+\xi_n^{p_1+1}) +$$

$$\left(\mu_n(\cdot)+\mu_n(\cdot)\delta+\frac{\lambda_n(\cdot)+\tau_n(\cdot)\delta}{2}\right)|\xi_n|^{1-p_n+1}\,|x_{n+1}^{p_n}-x_{n+1}^{*\,p_n}|+(\Psi_n(\cdot)-\dot{\hat{\Theta}})(\tilde{\Theta}+\eta_n(\cdot)), \tag{20}$$

所以, 鲁棒自适应控制器为:

$$\dot{\hat{\Theta}}=\Psi_n(\xi_1,\dots,\xi_n,\hat{\Theta},t),\quad u=u^*(\xi_1,\dots,\xi_n,\hat{\Theta}).$$

令 $n\beta=\gamma^2$, 可得:

$$\dot V_n(\xi_1,\dots,\xi_n,\tilde{\Theta})+y^{2mp_1}-\gamma^2\omega^{2m}\leqslant-(\xi_1^{p_1+1}+\dots+\xi_n^{p_1+1}). \tag{21}$$

显然, 系统(1)全局渐近稳定. 由于 $V_n(\cdot)$ 正定, 可从式(21)得到:

$$\int_0^t|y(s)|^{2mp_1}\mathrm{d}s\leqslant\gamma^2\int_0^t\omega^{2m}\mathrm{d}s. \tag{22}$$

定理 1的证明完成.

3 数值例子

在本部分, 给出了一个简单可行的具有光滑反馈控制器的非线性系统:

$$\begin{aligned}
\dot x_1&=x_1^2\left(1+x_2^{\frac{2}{3}}\right)(2+\sin^2\theta_1)x_2+x_1(1+x_2^2)^{\frac13}\theta_1^{x_1}+x_1(1+x_2^2)^{\frac13}\theta_1^{x_1}\omega,\ \theta_1\geqslant0,\\
\dot x_2&=x_1^2\left(1+x_2^{\frac{2}{3}}\right)(2+\sin^2\theta_1)u+\ln(1+(\theta_2x_2)^2)+\ln(1+(\theta_2x_2)^2)\omega,\ \theta_2\in\mathbf{R},\\
y&=x_1,
\end{aligned} \tag{23}$$

易证明下面的不等式成立:

$$\begin{gathered}
|\phi_1(x,\theta)|\leqslant|x_1||\theta_1^{x_1}|+\frac23|x_2|+\frac23|x_1^3\theta_1^{3x_1}|\leqslant\frac23|x_2|+|x_1|\left(1+\frac13x_1^2\right)e^{\frac{x_1^2}{2}}e^{\frac{9\ln^2\theta_1}{2}},\\
g_1(x,\theta)\leqslant\frac23|x_2|+|x_1|\left(1+\frac13x_1^2\right)e^{\frac{x_1^2}{2}}e^{\frac{9\ln^2\theta_1}{2}},\\
\lambda_1(x_1,x_2)=\tau_1(x_1,x_2)=2x_1^2\left(1+x_2^{\frac23}\right)\leqslant d_1(x_1,x_2,\theta_1)=d_2(x_1,x_2,\theta_1)\leqslant\\
\mu_1(x_1,x_2)=\mu_2(x_1,x_2)=3x_1^2\left(1+x_2^{\frac23}\right).
\end{gathered} \tag{24}$$

利用中值定理, 有 $|\phi_2(x,\theta)|\leqslant|\theta_2||x_2|$, $g_2(x,\theta)\leqslant|\theta_2||x_2|$.

选取 $\tilde{\Theta}=\Theta-\hat{\Theta}$, $\Theta=|\theta_2|+e^{\frac{9\ln^2\theta_1}{2}}$, $V_1(x_1,\tilde{\Theta})=\frac12x_1^2+\frac12\tilde{\Theta}^2$. 进而,

$$\begin{gathered}
\dot V_1+x_1^2-\omega^2\leqslant-2x_1^2+\left[\mu_1(x_1,x_2)+\mu_1(x_1,x_2)\delta+\frac{\lambda_1(x_1,x_2)+\tau_1(x_1,x_2)\delta}{2}\right]|x_1||x_2^{p_1}-x_2^{*\,p_1}|+\\
(\Psi_1(x_1)-\dot{\hat{\Theta}})(\tilde{\Theta}+\eta_1),
\end{gathered} \tag{25}$$

$$\Psi_1(x_1)=|x_1|^2\left(1+\frac13x_1^2\right)e^{\frac{x_1^2}{2}}(1+\delta),$$

$$x_2^*=-x_1\left[\frac{1+\left(1+\frac13x_1^2\right)e^{\frac{x_1^2}{2}}\sqrt{1+\hat{\Theta}^2}\left(1+\frac14x_1^2\beta_1(\cdot)\sqrt{1+\hat{\Theta}^2}\right)}{\lambda_1(x_1,x_2)+\tau_1(x_1,x_2)\delta}\right]=-x_1\alpha_1(x_1),$$

考虑 Lyapunov函数 $V_2(x_1,x_2,\tilde{\Theta})=V_1(x_1,\tilde{\Theta})+\frac12\xi_2^2$.

根据本文的方法, 设计如下光滑自适应控制器:

$$\dot{\hat{\Theta}}=x_1^2\left(1+\frac13x_1^2\right)e^{\frac{x_1^2}{2}}+\frac{x_1^2}{2+2\hat{\Theta}^2}+(x_2^{p_1}-x_2^{*\,p_1})^2\rho_2(\cdot),$$

$$u=-(x_2-x_2^*)\left(\frac{\frac{34}{9}+\rho_1(\cdot)+\rho_2(\cdot)\sqrt{1+\hat{\Theta}^2}+\rho_3(\cdot)+\rho_4(\cdot)}{\lambda_2(x_1,x_2)+\tau_2(x_1,x_2)\delta}\right),$$

其中

$$\beta_1(\cdot)=\left(1+\frac13x_1^2\right)e^{\frac{x_1^2}{2}},$$

$$\rho_1(\cdot) = \frac{5}{3}\left|\frac{\partial x_2^*}{\partial x_1}\right| + \left(\frac{5}{3}\frac{\partial x_2^*}{\partial x_1}\beta_1(\cdot)\right)^2,$$

$$\rho_2(\cdot) = (\delta+1)\left[1 + \frac{1+\hat{\Theta}^2}{2}\left(\beta_1(\cdot) + \left|\frac{\partial x_2^*}{\partial x_1}\right|\left(1+\frac{1}{3}x_1^2\right)e^{\frac{x_1^2}{2}}\right)^2\right],$$

$$\rho_3(\cdot) = \left(x_1^2\left(1+\frac{1}{3}x_1^2\right)e^{\frac{x_1^2}{2}} + \frac{x_1^2}{2+2\hat{\Theta}^2}\right)^2(3+x_1^2)^2 e^{\frac{x_1^2}{2}} + \rho_2(\cdot)\sqrt{1+x_1^2}\sqrt{1+(x_2-x_2^*)^2}(3+x_1^2)e^{\frac{x_1^2}{2}},$$

$$\rho_4(\cdot) = \delta^2\rho_2^2(\cdot)(1+\hat{\Theta}^2)^2(x_2-x_2^*)^2,$$

满足: $\dot{V}_2 + y^2 - 2\omega^2 \leqslant -(\xi_1^2+\xi_2^2) \leqslant 0$

显然, 系统(23)全局渐近稳定.

4 结论

本文利用 Backstepping方法, 研究了一类非线性不确定参数系统的鲁棒H_∞自适应控制问题. 通过巧妙设计动态反馈律, 解决了鲁棒H_∞干扰衰减问题.

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