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Merton推广模型的算术平均亚式期权定价

陈　波[1], 刘国祥[2], 石燕燕[3]

(1. 江苏教育学院数学与信息技术学院, 江苏 南京 210013)
(2. 南京师范大学数学科学学院, 江苏 南京 210097)
(3. 富登投资信用担保有限公司, 江苏 南京 210008)

[摘要] 假设金融资产价格服从 L vy过程且波动率是随机的, 利用鞅方法、测度变换以及 L vy过程的方法, 得到具有固定敲定价格的算术平均亚式看涨期权在任何有效时刻的价格公式.

[关键词] Merton模型, 亚式期权定价, L vy过程, 随机波动率
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Pricing of Arithmetic Average Asian Options Based on Generalized Merton Model

Chen Bo[1], Liu Guoxiang[2], Shi Yanyan[3]

(1. School of Mathematics and Information Technology, Jiangsu Institute of Education, Nanjing 210013, China)
(2. School of Mathematical Sciences, Nanjing Normal University, Nanjing 210097, China)
(3. Fullerton Investment & Credit Guarantee Company, Nanjing 210008, China)

Abstract: In this paper, we assumed the price of the financial asset is a L vy processes and the volatility is stochastic. By using the method of martingale, the transform of the measure, and L vy processes, we conduce the pricing of the call arithmetic average asian option at any valid time when the strike pricing is fixed.

Key words: Merton model, Asian option pricing, L vy processes, stochastic volatility

由于亚式看涨期权在金融领域的广泛应用, 对其定价的研究发展迅速, 很多学者作了大量的工作, 得到了许多成果[1-6], 但对标的资产的假设苛刻, 本文将讨论 Merton推广模型下亚式期权的定价.

1　Merton推广模型

给定流概率空间 $(\Omega, F, (F_t)_{0\leqslant t\leqslant T}, P)$. 设风险资产在时刻 t的价格为 S_t, 满足

$$S_t = S_0 e^{rt}\exp\left\{ a\sigma B_t^Q - \frac{1}{2}a^2\sigma^2 t + \int_c^\infty \ln(1+ax)(\mathrm{d}t, \mathrm{d}x) + \int_c^\infty [\ln(1-ax) - ax]\nu_Q(\mathrm{d}t, \mathrm{d}x)\right\}, \quad (1)$$

其中 $c = -a^{-1}$, Q是 (Ω, F)上的一个与测度 P等价的测度, $B_t^Q = B_t - \int_0^t F(s)\mathrm{d}s$, 是 Q-Brown运动,

$$\nu_Q(t, A) = \int_0^t\int_A (e^{H(s, x)} - 1)\nu(\mathrm{d}x)\mathrm{d}s,$$

$(t, A) = (t, A) - \nu_Q(t, A)$, 是 Q-鞅.

在上述模型中, 波动率 σ是常数, 而一般情况下波动率是变化的, 鉴于此, 我们可以作一些推广. 假设 λ是 $(R_{++}, B(R_{++}))$上给定的测度, 满足 $\lambda(R_{++}) = 1$, $\int_0^\infty \sigma\lambda(\mathrm{d}\sigma) < \infty$, 由 (1)式, 再关于测度 λ取 σ平均作为风险资产价格 S_t^*, 即有:

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通讯联系人: 陈　波, 讲师, 研究方向: 权证定价理论及实证. E-mail: pbchb@yahoo.com.cn

$$S_t^* = \int S_t \lambda(\mathrm{d}\sigma),$$

或

$$S_t^* = S_0^* e^{rt} \int \exp\{a\sigma B_t^Q - \frac{1}{2}a^2\sigma^2 t + \int^t \int \ln(1+ax)(\mathrm{d}t, \mathrm{d}x) + \int^t \int [\ln(1+ax) - ax]\nu(\mathrm{d}x)\mathrm{d}t\} \lambda(\mathrm{d}\sigma), \tag{2}$$

其中, $S_0^* = S_0$.

从而风险资产的折现价格为

$$S_t^* = S_0^* e^{rt} \int \exp\{a\sigma B_t - \frac{1}{2}a^2\sigma^2 t + \int^t \int \ln(1+ax)(\mathrm{d}t, \mathrm{d}x) + \int^t \int [\ln(1+ax) - ax]v(\mathrm{d}x)\mathrm{d}t\} \lambda(\mathrm{d}\sigma) = \int S_t \lambda(\mathrm{d}\sigma). \tag{3}$$

2　有关资产价格模型的一些结论

命题 1　$\{S_t^*\}_{0\leqslant t\leqslant T}$ 是 Q-鞅.

证明　因为 $\{S_t^*\}_{0\leqslant t\leqslant T}$ 是 Q-鞅, 由条件 Fubini定理

$$E^Q[S_t^* \mid F_t] = E^Q\left(\int \lambda(\mathrm{d}\sigma) \mid F_t\right) = \int E^Q(S_t \mid F_t)\lambda(\mathrm{d}\sigma) = \int \lambda(\mathrm{d}\sigma) = S_s^*.$$

命题 2　对于 $\forall 0 \leqslant s < t \leqslant T$,

$$S_t^* = S_s^* e^{r(t-s)} \int \exp(Z_t - Z_s)\lambda_s(\mathrm{d}\sigma),\quad a.\ s \tag{4}$$

其中:

$$Z_t = a\sigma B_t^Q - \frac{1}{2}a^2\sigma^2 t + \int^t \int \ln(1+ax)N_Q(\mathrm{d}t, \mathrm{d}x) + \int^t \int [\ln(1+ax) - ax]\nu_Q(\mathrm{d}x)\mathrm{d}t\lambda(\mathrm{d}\sigma),$$

$$\lambda_t(\mathrm{d}\sigma) = \frac{\exp\left\{a\sigma B_t^Q - \frac{1}{2}a^2\sigma^2 t\right\}}{\int \exp\left\{a\sigma B_t^Q - \frac{1}{2}a^2\sigma^2 t\right\}\lambda(\mathrm{d}\sigma)}\lambda(\mathrm{d}\sigma).$$

证明　记

$$Z_t^1 = \int^t \int \ln(1+ax)(\mathrm{d}t, \mathrm{d}x) + \int^t \int [\ln(1+ax) - ax]\nu_Q(\mathrm{d}x)\mathrm{d}t,$$

则

$$S_t^* = S_0^* e^{rt} \int \exp\left\{a\sigma B_t^Q - \frac{a^2\sigma^2}{2}t\right\}\lambda(\mathrm{d}\sigma)e^{Z_t^1} =$$

$$\frac{S_s^* S_0^* e^{rt} \int \exp\left\{a\sigma B_t^Q - \frac{a^2\sigma^2}{2}t\right\}\lambda(\mathrm{d}\sigma)e^{Z_t^1}}{S_s^*} = \frac{S_s^* e^{rt} \int \exp\left\{a\sigma B_t^Q - \frac{a^2\sigma^2}{2}t\right\}\lambda(d\sigma)e^{Z_t^1}}{e^{rs}\int \exp\left\{a\sigma B_s^Q - \frac{a^2\sigma^2}{2}s\right\}\lambda(\mathrm{d}\sigma)e^{Z_s^1}} =$$

$$S_s^* e^{r(t-s)} \int \exp\left\{a\sigma(B_t^Q - B_s^Q) - \frac{1}{2}a^2\sigma^2(t-s)\right\}\cdot \frac{\exp\left\{a\sigma B_s^Q - \frac{1}{2}a^2\sigma^2 s\right\}}{\int \exp\left\{a\sigma B_s^Q - \frac{1}{2}a^2\sigma^2 s\right\}\lambda(\mathrm{d}\sigma)}\lambda(\mathrm{d}\sigma)e^{Z_t^1 - Z_s^1} =$$

$$S_s^* e^{r(t-s)} \int \exp\left\{a\sigma(B_t^Q - B_s^Q) - \frac{1}{2}a^2\sigma^2(t-s)\right\}\lambda_s(\mathrm{d}\sigma)e^{Z_t^1 - Z_s^1} = S_s^* e^{r(t-s)} \int \exp(Z_t - Z_s)\lambda_s(\mathrm{d}\sigma).$$

3　算术平均亚式期权的定价公式

下面讨论亚式看涨期权的价格, 其在到期日 T 的值依赖于直到时间 T 价格过程整个路径的均值:

$\frac{1}{T}\int_0^T S_u\,\mathrm{d}u$, 由于在新测度 Q 下, 股票折现价格为一鞅, 这样敲定价为 K 算术平均亚式上涨期权的价格由折现条件期望给出.

$$C_F^*(K,t) = \mathrm{e}^{-r(T-t)}E^Q\left[\left(\frac{1}{T}\int_0^T S_u^*\,\mathrm{d}u - K\right)_+ \mid F_t\right], \tag{5}$$

其中 $C_F^*(K,t)$ 表示敲定价为 K 的算术平均亚式看涨期权在时刻 t 的价格, $E^Q(\cdot)$ 表示在概率测度 Q 下求期望.

命题 3　记 $q^* = \dfrac{KT - \int_0^t S_u^*\,\mathrm{d}t}{S_u^*}$, 则

$$C_F^*(K,t) = \frac{1}{T}\mathrm{e}^{-r(T-t)}S_t^*E^Q\left[\int_0^{T-t}\mathrm{e}^{ru}\int_0^\infty \mathrm{e}^{Z_u}\lambda_t(\mathrm{d}\sigma)\,\mathrm{d}u - q^*\right]_+. \tag{6}$$

证明

$$C_F^*(K,t) = \mathrm{e}^{-r(T-t)}E^Q\left[\left(\frac{1}{T}\int_0^T S_u^*\,\mathrm{d}u - K\right)_+ \mid F_t\right] =$$
$$\frac{1}{T}\mathrm{e}^{-r(T-t)}E^Q\left[\left(\int_t^T S_u^*\,\mathrm{d}u - \left(TK - \int_0^t S_u^*\,\mathrm{d}u\right)\right)_+ \mid F_t\right],$$

代入(4)式, 并注意到 Z_t 的独立增量性和平稳增量性, 以及 $\lambda_t(\mathrm{d}\sigma)$ 关于 F_t 可测, 于是得

$$C_F^*(K,t) = \frac{1}{T}\mathrm{e}^{-r(T-t)}E^Q\left[\int_t^T S_t^*\,\mathrm{e}^{r(u-t)}\int_0^\infty \exp(Z_u - Z_t)\lambda_t(\mathrm{d}\sigma)\,\mathrm{d}u - \left(TK - \int_0^t S_u^*\,\mathrm{d}u\right)\right]_+ =$$
$$\frac{1}{T}\mathrm{e}^{-r(T-t)}S_t^*E^Q\left[\int_0^{T-t}\mathrm{e}^{ru}\int_0^\infty \exp(Z_u - Z_s)\lambda_t(\mathrm{d}\sigma)\,\mathrm{d}u - \frac{\left(TK - \int_0^t S_u^*\,\mathrm{d}u\right)}{S_t^*}\right]_+ =$$
$$\frac{1}{T}\mathrm{e}^{-r(T-t)}S_t^*E^Q\left[\int_0^{T-t}\mathrm{e}^{ru}\int_0^\infty \mathrm{e}^{Z_u}\lambda_t(\mathrm{d}\sigma)\,\mathrm{d}u - q^*\right]_+.$$

下面我们推导 $q^* \mid \mathscr{F}_t \leqslant 0$ 时 $C_F^*(K,t)$ 的精确值, 该值与我们所选的测度 Q 无关; 但是, 当 $q^* \mid \mathscr{F}_t > 0$ 时, 我们能得到 $C_F^*(K,t)$ 的所谓上界, 但它与测度 Q 有关.

定理 1　在 t 时刻, 若 $q^* \leqslant 0$, 则

$$C_F^*(K,t) = \frac{S_t^*}{rT} - \left[\frac{S_t^*}{rT} - K + \frac{1}{T}\int_0^t S_u^*\,\mathrm{d}u\right]\mathrm{e}^{-r(T-t)} =: A_1^*. \tag{7}$$

证明　由命题 1知, $\{S_t^*\}_{0\leqslant t\leqslant T}$ 是 Q-鞅, 又 $S_t^* = S_0\mathrm{e}^{Z_t}$, 所以 $\{\mathrm{e}^{Z_t}\}_{0\leqslant t\leqslant T}$ 也是 Q-鞅. 又 $q^* \mid \mathscr{F}_t \leqslant 0$, 所以

$$\int_0^{T-t}\mathrm{e}^{ru}\int_0^\infty \mathrm{e}^{Z_u}\lambda_t(\mathrm{d}\sigma)\,\mathrm{d}u - q^* \geqslant 0$$

于是由(6)式,

$$C_F^*(K,t) = \frac{1}{T}\mathrm{e}^{-r(T-t)}S_t^*E^Q\left[\int_0^{T-t}\mathrm{e}^{ru}\int_0^\infty \mathrm{e}^{Z_u}\lambda_t(\mathrm{d}\sigma)\,\mathrm{d}u - q^*\right] =$$
$$\frac{1}{T}\mathrm{e}^{-r(T-t)}S_t^*\left[\int_0^{T-t}\mathrm{e}^{ru}\int_0^\infty E^Q\mathrm{e}^{Z_u}\lambda_t(\mathrm{d}\sigma)\,\mathrm{d}u - q^*\right] =$$
$$\frac{1}{T}\mathrm{e}^{-r(T-t)}S_t^*\left[\int_0^{T-t}\mathrm{e}^{ru} - q^*\right] = \frac{S_t^*}{rT} - \left[\frac{S_t^*}{rT} - K + \frac{1}{T}\int_0^t S_u^*\,\mathrm{d}u\right]\mathrm{e}^{-r(T-t)}.$$

命题 4　$\left\{\mathrm{e}^{\int_0^t G^*(t)\mathrm{d}t + 2a\sigma B_u^Q + 2\int_0^t\int_c^\infty \ln(1+ax)(\mathrm{d}t,\,\mathrm{d}x)}\right\}$ 是 Q-鞅的充要条件为

$$G^*(u) + 2a^2\sigma^2 + \int[(1+ax)^2 - 1 - 2\ln(1+ax)](\mathrm{e}^{H(u,x)} - 1)\nu(\mathrm{d}x) = 0, \quad a.\ s$$

证明　考察 $\mathrm{e}^{Y_u^*}$, 其中 Y_u^* 满足

$$\mathrm{d}Y_u^* = G^*(u)\,\mathrm{d}u + 2a\sigma\,\mathrm{d}B_u^Q + 2\int \ln(1+ax)N_Q(\mathrm{d}u,\,\mathrm{d}x),$$

则由 Itô公式, 并注意到

$$\nu_Q(\mathrm{d}x,\mathrm{d}s)=(\mathrm{e}^{H(s,x)}-1)\nu(\mathrm{d}x)\mathrm{d}s,$$

$$\begin{aligned}\mathrm{e}^{Y_u^*}&=1+\int_0^u \mathrm{e}^{Y_{s-}^*}2a\sigma\mathrm{d}B_s^Q+\int_0^u\int_0^\infty \mathrm{e}^{Y_{s-}^*}(\mathrm{e}^{\ln(1+ax)}-1)N_Q(\mathrm{d}x,\mathrm{d}s)+\\&\int_0^u \mathrm{e}^{Y_{s-}^*}[G^*(s)+2a^2\sigma^2]\mathrm{d}s+\\&\int_0^u\int_0^\infty \mathrm{e}^{Y_{s-}^*}[\mathrm{e}^{2\ln(1+ax)}-1-2\ln(1+ax)]\nu_Q(\mathrm{d}x,\mathrm{d}s)=\\&1+\int_0^u \mathrm{e}^{Y_{s-}^*}2a\sigma\mathrm{d}B_s^Q+\int_0^u\int_0^\infty \mathrm{e}^{Y_{s-}^*}(\mathrm{e}^{\ln(1+ax)}-1)N_Q(\mathrm{d}x,\mathrm{d}s)+\int_0^u \mathrm{e}^{Y_{s-}^*}[G^*(s)+2a^2\sigma^2]\mathrm{d}s+\\&\int_0^u\int_0^\infty \mathrm{e}^{Y_{s-}^*}[\mathrm{e}^{2\ln(1+ax)}-1-2\ln(1+ax)](\mathrm{e}^{H(s,x)}-1)\nu(\mathrm{d}x)\mathrm{d}s=\\&1+\int_0^u \mathrm{e}^{Y_{s-}^*}2a\sigma\mathrm{d}B_s^Q+\int_0^u\int_0^\infty \mathrm{e}^{Y_{s-}^*}(\mathrm{e}^{\ln(1+ax)}-1)N_Q(\mathrm{d}x,\mathrm{d}s)+\\&\int_0^u \mathrm{e}^{Y_{s-}^*}\left\{G^*(s)+2a^2\sigma^2+\int_0^\infty[(1+ax)^2-1-2\ln(1+ax)](\mathrm{e}^{H(s,x)}-1)\nu(\mathrm{d}x)\right\}\mathrm{d}s,\end{aligned}$$

于是, $\{\mathrm{e}^{Y_u^*}\}$ 是鞅的充分必要条件是

$$G^*(u)+2a^2\sigma^2+\int_0^\infty[(1+ax)^2-1-2\ln(1+ax)](\mathrm{e}^{H(u,x)}-1)\nu(\mathrm{d}x)=0.$$

命题 5 设在所作的等价鞅测度变换中, $H(s,x)$ 为非随机的, 则

$$E^Q\mathrm{e}^{2Z_u}=\exp\left\{a^2\sigma^2u+a^2\int_0^u\int_0^\infty x^2[\mathrm{e}^{H(t,x)}-1]\nu(\mathrm{d}x)\mathrm{d}t\right\}.\tag{8}$$

证明　利用命题 4的结果, 设 $G^*(u)$ 满足

$$\int_0^u G^*(t)\mathrm{d}t=-2a^2\sigma^2u-\int_0^u\int_0^\infty[(1+ax)^2-1-2\ln(1+ax)]\nu_Q(\mathrm{d}x,\mathrm{d}t),$$

则

$$\begin{aligned}E^Q\mathrm{e}^{2Z_u}&=E^Q\exp\left\{2a\sigma B_u^Q-a^2\sigma^2u+2\int_0^u\int_0^\infty\ln(1+ax)N_Q(\mathrm{d}t,\mathrm{d}x)+\right.\\&\left.2\int_0^u\int_0^\infty[\ln(1+ax)-ax]\nu_Q(\mathrm{d}x,\mathrm{d}t)\right\}=\\&E^Q\exp\left\{\int_0^u G^*(t)\mathrm{d}t+2a\sigma B_u^Q+2\int_0^u\int_0^\infty\ln(1+ax)(\mathrm{d}t,\mathrm{d}x)\right\}\times\\&\exp\left\{-\int_0^u G^*(t)\mathrm{d}t-a^2\sigma^2u+2\int_0^u\int_0^\infty[\ln(1+ax)-ax]\nu_Q(\mathrm{d}x,\mathrm{d}t)\right\}=\\&E^Q[\mathrm{e}^{Y_u^*}]\exp\left\{-\int_0^u G^*(t)\mathrm{d}t-a^2\sigma^2u+2\int_0^u\int_0^\infty[\ln(1+ax)-ax]\nu_Q(\mathrm{d}x,\mathrm{d}t)\right\}=\\&\exp\left\{-\int_0^u G^*(t)\mathrm{d}t-a^2\sigma^2u+2\int_0^u\int_0^\infty[\ln(1+ax)-ax]\nu_Q(\mathrm{d}x)\mathrm{d}t\right\}=\\&\exp\left\{a^2\sigma^2u+\int_0^u\int_0^\infty a^2x^2[\mathrm{e}^{H(t,x)}-1]\nu(\mathrm{d}x)\mathrm{d}t\right\}.\end{aligned}$$

定理 2 设在所作的等价鞅测度变换中, $H(s,x)$ 为非随机的. 若在 t时刻, $q^*>0$, 则

$$\begin{aligned}C_F^*(K,t)\leqslant\frac{1}{2}A_1^*+\frac{S_t^*}{2T}\mathrm{e}^{-r(T-t)}&\left\{\int_0^{T-t}\int_0^\infty\exp\left[2ru+a^2\sigma^2u+\right.\right.\\&\left.\left.a^2\int_0^u\int_0^\infty x^2(\mathrm{e}^{H(s,x)}-1)\nu(\mathrm{d}x)\mathrm{d}s\right]\lambda_t(\mathrm{d}\sigma)\mathrm{d}u-\frac{2q^*}{r(T-t)}[\mathrm{e}^{r(T-t)}-1]+\frac{q^{*2}}{T-t}\right\}^{\frac{1}{2}},\end{aligned}\tag{9}$$

其中 A_1^* 由(7)式所示.

证明　对于任何实数 a,b, 有 $(a-b)_+=\frac{1}{2}(a-b)+\frac{1}{2}|a-b|$, 于是由(6)式

$$C_F^*(K,t)=\frac{S_t^*}{T}\mathrm{e}^{-r(T-t)}E^Q\left[\left(\int_0^{T-t}\mathrm{e}^{ru}\int_0^\infty\mathrm{e}^{Z_u}\lambda_t(\mathrm{d}\sigma)\mathrm{d}u-q^*\right)_+\right]=$$

$$\frac{S_t^*}{T}e^{-r(T-t)}E^Q\left[\frac{1}{2}\left(\int_t^T e^{ru}\int_0^\infty e^{Z_u}\lambda_t(\mathrm{d}\sigma)\,\mathrm{d}u-q^*\right)+\frac{1}{2}\left|\int_t^T e^{ru}\int_0^\infty e^{Z_u}\lambda_t(\mathrm{d}\sigma)\,\mathrm{d}u-q^*\right|\right]\leqslant$$

$$\frac{S_t^*}{2T}e^{-r(T-t)}E^Q\left[\int_t^T\int_0^\infty\left(e^{ru}e^{Z_u}-\frac{q^*}{T-t}\right)\lambda_t(\mathrm{d}\sigma)\,\mathrm{d}u\right]+\frac{S_t^*}{2T}e^{-r(T-t)}E^Q\left[\int_t^T\int_0^\infty\left|e^{ru}e^{Z_u}-\frac{q^*}{T-t}\right|\lambda_t(\mathrm{d}\sigma)\,\mathrm{d}u\right]\leqslant$$

$$\frac{A_1^*}{2}+\frac{1}{2T}e^{-r(T-t)}S_t^*\left\{\int_t^T\int_0^\infty E^Q\left[e^{ru}e^{Z_u}-\frac{q^*}{T-t}\right]^2\lambda_t(\mathrm{d}\sigma)\,\mathrm{d}u\right\}^{\frac{1}{2}}=$$

$$\frac{1}{2}A_1^*+\frac{S_t^*}{2T}e^{-r(T-t)}\left\{\int_t^T\int_0^\infty E^Q\left[e^{2ru}e^{2Z_u}-\frac{2q^*}{T-t}e^{ru}e^{Z_u}+\left(\frac{q^*}{T-t}\right)^2\right]\lambda_t(\mathrm{d}\sigma)\,\mathrm{d}u\right\}^{\frac{1}{2}}=$$

$$\frac{1}{2}A_1^*+\frac{S_t^*}{2T}e^{-r(T-t)}\left\{\int_t^T\int_0^\infty\left[e^{2ru}E^Q e^{2Z_u}-\frac{2q^*}{T-t}e^{ru}E^Q e^{Z_u}+\left(\frac{q^*}{T-t}\right)^2\right]\lambda_t(\mathrm{d}\sigma)\,\mathrm{d}u\right\}^{\frac{1}{2}}=$$

$$\frac{1}{2}A_1^*+\frac{S_t^*}{2T}e^{-r(T-t)}\left\{\int_t^T\int_0^\infty\left[\exp\left(2ru+a^2\sigma^2u+a^2\int_0^u\int x^2(e^{H(s,x)}-1)\nu(\mathrm{d}x)\,\mathrm{d}s\right)-\right.\right.$$
$$\left.\left.\frac{2q^*}{T-t}e^{ru}+\left(\frac{q^*}{T-t}\right)^2\right]\lambda_t(\mathrm{d}\sigma)\,\mathrm{d}u\right\}^{\frac{1}{2}}=\frac{1}{2}A_1^*+\frac{S_t^*}{2T}e^{-r(T-t)}\left\{\int_t^T\int_0^\infty\exp\left[2ru+a^2\sigma^2u+\right.\right.$$
$$\left.\left.a^2\int_0^u\int x^2(e^{H(s,x)}-1)\nu(\mathrm{d}x)\,\mathrm{d}s\right]\lambda_t(\mathrm{d}\sigma)\,\mathrm{d}u-\frac{2q^*}{r(T-t)}[e^{r(T-t)}-1]+\left(\frac{q^*}{T-t}\right)^2\right\}^{\frac{1}{2}}.$$

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