一类基于观测器法的非线性系统的输出反馈控制

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[摘要] 研究了一类非严格三角形式非线性系统的输出反馈控制问题. 结合观测器设计法, 技巧性地选取 Lyapunov函数, 构造了一个动态输出补偿器和光滑控制器, 使系统达到全局渐近稳定. 扩展了系统结构, 弱化了结论成立的条件, 最后的仿真例子阐明了结论的正确性.

[关键词] 状态观测器, 输出反馈控制, 全局渐近稳定

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Output Feedback Control of a Class of Nonlinear Systems Using Observer Design Method

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Abstract: This paper studies the problem about the output feedback control of nonlinear systems with non-strict triangular form. Using the normal observer design method, a Lyapunov function is chosen and a dynamic output compensator is constructed, which guarantees the systematic globally asymptotic stable. The form of nonlinear systems is developed and the conclusion implements the current results. Finally, the example illustrates the correctness of the conclusion.

Key words: state observer, output feedback control, globally asymptotic stable

本文研究如下形式的一类非线性系统

$$\begin{aligned} \dot{x}_i &= \sigma_i(t, x_1, x_2, \cdots, x_i) + \tau_i(x_1)x_{i+1}, \\ \dot{x}_n &= \sigma_n(t, x_1, x_2, \cdots, x_n) + \tau_n(x_1)u, \\ y &= x_1, \end{aligned} \tag{1}$$

其中, $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)^{\mathrm{T}} \in R$, $u \in R$, $y \in R$分别是系统状态, 控制输入, 系统输出. 对 $i = 1, n$, $\tau_i(\cdot) > 0$, $\sigma_i(\cdot)$是连续函数.

过去十几年, 非线性系统输出反馈的全局渐近稳定问题一直是控制领域研究热点之一. 经过不懈努力, 对 $\tau_i(\cdot)$, $\sigma_i(\cdot)$施加适当条件, 研究者得到了一些重要结论[1-3]. 在线性增长条件下, 文献[4]设计了光滑反馈控制器和齐观测器, 此种输出反馈设计算法相对于文献[5]具有较强的鲁棒性. 文献[6]在系统满足线性增长条件下, 利用输出反馈控制法精确建立了一个线性输出补偿器使闭环系统全局指数稳定. 依据单输出非线性系统观测器是一致可观测和可全局 Lipschitz化的结论, 文献[7]设计了半全局有限时间收敛的观测器, 进而得到一种使系统局部有限时间稳定的方法. 一般情况下, 解决非线性系统输出反馈的稳定问题常用方法是: 设计状态观测器(Lunberge形式等)和补偿器, 定义动态误差, 取定 Lyapunov函数, 构造适当的控制器使系统稳定, 如[8-10]. 所研究的非线性系统多以 $\tau_i(\cdot)$是常数、系统形式是严格下三角的且满足线性增长条件或扰动项有界为主.

本文所考察的非线性系统为非严格三角形式, 不满足线性增长条件, $\tau_i(\cdot)$是实值函数. 目前, 利用观

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测器法来解决此类系统输出反馈稳定问题的结论较少. 相对于已经获得的方法, 本文结论具有简单、易进一步推广、应用范围广的优点.

1 主要结果

为证明方便, 首先引入下面的假定和引理.

假定 1 $\sigma_i(\cdot)=\sum_{j=1}^{i}a_j(x_1)x_j$, 其中, $a_j(x_1)>0$为连续函数.

引理 1 假设 m 和 n 是两个正实数, a, b, $\pi(\boldsymbol{x}, y)$ 是连续正值函数, 则对任意常数 $c\geqslant 0$有

$$|a|^n|b|^m\pi(\boldsymbol{x}, y)\leqslant c|a|^{m+n}+\frac{m}{n+m}\left(\frac{n}{c(n+m)}\right)^{\frac{n}{m}}|b|^{m+n}\pi(\boldsymbol{x}, y)^{m+n}.$$

引理 2 存在如下形式光滑状态反馈控制器

$$\begin{aligned}&u^*=-b_n\xi_n(\boldsymbol{x}),\\&\xi_n(\boldsymbol{x})=x_n+b_{n-1}x_{n-1}+b_2x_2+b_1x_1,\end{aligned}\tag{2}$$

使

$$\dot{W}|_{(1)-(2)}\leqslant-\sum_{i=1}^{n}\xi_i^2+\tau_n(x_1)\xi_n(u-u^*),\tag{3}$$

其中,

$$\begin{aligned}&x_1^*=0, \xi_1=x_1-x_1^*, x_2^*=-\xi_1\beta_1(x_1), \xi_2=x_2-x_2^*,\\&\cdots\\&x_n^*=-\xi_{n-1}\beta_{n-1}(x_1, x_2, \cdots, x_{n-1}), \xi_n=x_n-x_n^*,\end{aligned}\tag{4}$$

$b_i(1\leqslant i\leqslant n)$ 是实数, $W(\boldsymbol{x})$ 是正定适当的 Lyapunov函数.

证明 选取正定适当的 Lyapunov函数 (形如文献[10])

$$W(x_1, x_2, \cdots, x_n)=\sum_{i=1}^{n}W_i(x_1, x_2, \cdots, x_i)=\sum_{i=1}^{n}\int_{x_i^*}^{x_i}(s-x_i^*)\mathrm{d}s,$$

借助假定 1和引理 1, 对 $W_k(\cdot)$ 沿系统(1)求导

$$\begin{aligned}\dot{W}_k=&\frac{\partial W_k}{\partial x_k}\dot{x}_k+\sum_{l=1}^{k-1}\frac{\partial W_k}{\partial x_l}\dot{x}_l\leqslant\\&\xi_k(\tau_k(x_1)x_{k+1}+\sigma_k(\cdot))+\sum_{l=1}^{k-1}\xi_k\left|\frac{\partial x_k^*}{\partial x_l}\right|(\tau_l(x_1)+\sigma_l(\cdot))\leqslant\\&\tau_k(x_1)\xi_k(x_{k+1}-x_{k+1}^*)+\tau_k(x_1)x_{k+1}^*+\xi_k\sum_{j=1}^{k}a_j(x_1)x_j+\\&\xi_k(|\xi_1|+|\xi_2|+\cdots+|\xi_k|)\sum_{i=1}^{k-1}h_i(x_1, x_2, \cdots, x_{k-1})\leqslant\\&\tau_k(x_1)\xi_k(x_{k+1}-x_{k+1}^*)+\tau_k(x_1)x_{k+1}^*+\sum_{i=1}^{k-1}\xi_i^2+\xi_k^2\rho_k(x_1, x_2, \cdots, x_{k-1}),\end{aligned}$$

其中, $h_i(\cdot)$, $\rho_k(\cdot)$ 为光滑正值函数.

取 $x_{k+1}^*=-\xi_k\left[\frac{n-k+1+\rho_k(\cdot)}{\tau_k(x_1)}\right]$, 可有

$$\dot{W}_k\leqslant-(n-k+1)\xi_k^2+\sum_{i=1}^{k-1}\xi_i^2+\tau_k(x_1)\xi_k(x_{k+1}-x_{k+1}^*),$$

因此, 可得

$$\begin{aligned}\dot{W}=&\dot{W}_1+\dot{W}_2+\cdots+\dot{W}_n\leqslant-n\xi_1^2+[-(n-1)\xi_2^2+\xi_1^2]+\\&\left[-(n-2)\xi_3^2+\sum_{i=1}^{2}\xi_i^2\right]+\cdots+\left[-\xi_n^2+\sum_{i=1}^{n-1}\xi_i^2\right]+\xi_n\tau_n(x_1)(u-u^*)=\\&-\sum_{i=1}^{n}\xi_i^2+\xi_n\tau_n(x_1)(u-u^*),\end{aligned}$$

引理得证. 以下证明本文的主要定理.

定理　存在适当常数 L_0, M, b_i, l_j, 当

$$c_i > p_0 > 0,\ q_k > p_0 > 0,\ i = 1, 2, \cdots, n,\ j = 2, 3, \cdots, n+1,\ k = 2, 3, \cdots, n$$

时 (其中, c_i, p_0, q_k 形如 (21)), 系统 (1) 全局渐近稳定.

证明　设计如下的状态观测器

$$\begin{aligned} \dot{\hat{z}}_{i+1} &= -l_i(\tau_i(x_1)\hat{x}_{i+1} + \sigma_i(t, x_1, \hat{x}_2, \cdots, \hat{x}_i)), \\ \hat{x}_{i+1} &= \hat{z}_{i+1} + l_i\hat{x}_i, \end{aligned} \tag{5}$$

同时, 有 $x_{i+1} = z_{i+1} + l_i x_i$. 取 $e_i = z_i - \hat{z}_i$, 误差动态可表为

$$\begin{aligned} \dot{e}_2 &= \tau_2(x_1)x_3 + \sigma_2(\cdot) - l_1[\tau_1(x_1)e_2 + \sigma_1(\cdot) - \sigma_1(\cdot)], \\ \dot{e}_3 &= \tau_3(x_1)x_4 + \sigma_3(\cdot) - l_2[\tau_2(x_1)(e_3 + l_2(x_2 - \hat{x}_2)) + \sigma_2(\cdot) - \sigma_2(\cdot)], \\ &\cdots \\ \dot{e}_n &= \tau_n(x_1)x_{n+1} + \sigma_n(\cdot) - l_{n-1}[\tau_{n-1}(x_1)(e_n + l_{n-1}(x_{n-1} - \hat{x}_{n-1})) + \sigma_{n-1}(\cdot) - \sigma_{n-1}(\cdot)], \end{aligned} \tag{6}$$

其中, $l_i > 0$, $\sigma_i(\cdot) = \sigma_i(t, x_1, \hat{x}_2, \cdots, \hat{x}_i)$, $i = 1, \cdots, n$, $u = x_{n+1}$.

选取正定适当的 Lyapunov函数

$$V(e_2, e_3, \cdots, e_n) = \frac{1}{2}(e_2^2 + e_3^2 + \cdots + e_n^2),$$

对 $V(\cdot)$ 沿系统 (1) 求导得

$$\begin{aligned} \dot{V} = &\sum_{i=2}^{n} e_i\tau_i(x_1)x_{i+1} + \sigma_i(\cdot) - l_{i-1}[\tau_{i-1}(x_1)(e_i + l_{i-1}(x_{i-1} - \hat{x}_{i-1})) + \sigma_{i-1}(\cdot) - \sigma_{i-1}(\cdot)] = \\ &\sum_{i=2}^{n} e_i\sigma_i(\cdot) + \sum_{i=2}^{n} e_i\tau_i(x_1)x_{i+1} - \sum_{i=2}^{n} l_{i-1}\tau_{i-1}(x_1)e_i^2 - \\ &\sum_{i=2}^{n} l_{i-1}^2\tau_{i-1}(x_1)(x_{i-1} - \hat{x}_{i-1})e_i - \sum_{i=2}^{n} l_{i-1}(\sigma_{i-1}(\cdot) - \sigma_{i-1}(\cdot))e_i, \end{aligned} \tag{7}$$

利用假定 1和引理 1, 直接计算得

$$\begin{aligned} \left|\sum_{i=2}^{n} e_i\sigma_i(\cdot)\right| &\leqslant \sum_{i=2}^{n}\left|e_i\sum_{j=1}^{i} a_j(x_1)x_j\right| \leqslant \frac{1}{2}\sum_{i=2}^{n}\sum_{j=1}^{i}\frac{1}{2}a_j(x_1)(e_i^2 + x_j^2), \\ \left|\sum_{i=2}^{n} e_i\tau_i(x_1)x_{i+1}\right| &\leqslant \frac{1}{2}\sum_{i=2}^{n}\tau_i(x_1)(e_i^2 + x_{i+1}^2). \end{aligned} \tag{8}$$

据观测器形式 (5), 有

$$\begin{aligned} |x_i - \hat{x}_i| &= |e_i + l_{i-1}(x_{i-1} - \hat{x}_{i-1})| = |e_i + l_{i-1}[e_{i-1} + l_{i-2}(x_{i-2} - \hat{x}_{i-2})]| = \\ &|e_i + l_{i-1}(e_{i-1} + \cdots + l_2e_2)\cdots)| \leqslant c\sum_{j=2}^{i} l_j l_{j-1}\cdots l_2|e_j|, \end{aligned} \tag{9}$$

其中, $c > 0$.

进而, 借助引理 1和 (9), 得到

$$\begin{aligned} &\left|\sum_{i=2}^{n} l_{i-1}^2\tau_{i-1}(x_1)(x_{i-1} - \hat{x}_{i-1})e_i\right| \leqslant c\sum_{i=2}^{n} l_{i-1}^2\tau_{i-1}(x_1)\sum_{k=2}^{i-1} l_k l_{k-1}\cdots l_2|e_k e_i| \leqslant \\ &\frac{1}{2}\sum_{i=2}^{n} l_{i-1}^2\tau_{i-1}(x_1)\sum_{j=2}^{i-1} c l_j l_{j-1}\cdots l_2(e_j^2 + e_i^2). \end{aligned} \tag{10}$$

根据假定 1和 (9), 得出

$$\begin{aligned} &\left|\sum_{i=2}^{n} l_{i-1}(\sigma_{i-1}(\cdot) - \sigma_{i-1}(\cdot))e_i\right| = \sum_{i=2}^{n} l_{i-1}e_i\sum_{j=2}^{i-1} a_j(x_1)(x_j - \hat{x}_j) \leqslant \\ &\sum_{i=2}^{n} l_{i-1}\sum_{j=2}^{i-1} a_j(x_1)\sum_{j=2}^{n-1} c l_j l_{j-1}\cdots l_2|e_j e_i| \leqslant \frac{1}{2}\sum_{i=2}^{n} l_{i-1}\sum_{j=1}^{i-1} a_j(x_1)\sum_{j=2}^{i-1} c l_j l_{j-1}\cdots l_2(e_j^2 + e_i^2). \end{aligned} \tag{11}$$

将 (8), (10) 和 (11) 代入 (7), 得

$$\dot{V} \leqslant \frac{1}{2}\sum_{i=2}^{n}\sum_{j=1}^{i}\frac{1}{2}a_j(x_1)(e_i^2 + x_j^2) + \frac{1}{2}\sum_{i=2}^{n}\tau_i(x_1)(e_i^2 + x_{i+1}^2) - \sum_{i=2}^{n} l_{i-1}\tau_{i-1}(x_1)e_i^2 +$$

$$\frac{1}{2}\sum_{i=2}^{n} l_{i-1}^{2}\tau_{i-1}(x_1)\sum_{j=2}^{i-1} l_j l_{j-1}\cdots l_2(e_j^2+e_i^2)+\frac{1}{2}\sum_{i=2}^{n} l_{i-1}\sum_{k=1}^{i-1} a_k(x_1)\sum_{j=2}^{i-1} c l_j l_{j-1}\cdots l_2(e_j^2+e_i^2)=$$

$$\begin{aligned}&\frac{1}{2}\sum_{i=2}^{n}\Big[\sum_{j=1}^{i} a_j(x_1)+\tau_i(x_1)-2l_{i-1}\tau_{i-1}(x_1)+l_{i-1}^{2}\tau_{i-1}(x_1)\sum_{j=2}^{i-1} l_j l_{j-1}\cdots l_2+\\&l_{i-1}\sum_{k=1}^{i-1} a_k(x_1)\sum_{j=2}^{i-1} c l_j l_{j-1}\cdots l_2\Big]e_i^2+\frac{1}{2}\sum_{i=2}^{n}\Big[l_{i-1}^{2}\tau_{i-1}(x_1)\sum_{j=2}^{i-1} l_j l_{j-1}\cdots l_2+\\&l_{i-1}\sum_{k=1}^{i-1} a_k(x_1)\sum_{j=2}^{i-1} c l_j l_{j-1}\cdots l_2\Big]e_i^2+\frac{1}{2}\sum_{i=2}^{n}\Big[\sum_{j=1}^{i} a_j(x_1)x_j^2+\tau_i(x_1)x_{i+1}^2\Big].\end{aligned}\tag{12}$$

下面, 考虑(4), 可有

$$\xi_n^2=(x_n+\xi_{n-1}\beta_{n-1}(\cdot))^2\leqslant 2\sum_{i=1}^{n-1} b_i x_i^2+2x_n^2,$$

$$\begin{aligned}\xi_n^2&=(x_n+\xi_{n-1}\beta_{n-1}(\cdot))^2=x_n^2+2\beta_{n-1}(\cdot)x_n\xi_{n-1}+\beta_{n-1}^2(\cdot)\xi_{n-1}^2=\\&\frac{1}{2}\xi_{n-1}^2+\frac{1}{2\beta_{n-1}^2(\cdot)+1}x_n^2+\left[\frac{\beta_{n-1}(\cdot)}{\sqrt{\beta_{n-1}^2(\cdot)+\frac{1}{2}}}x_n+\sqrt{\beta_{n-1}^2(\cdot)+\frac{1}{2}}\right]^2+\frac{1}{2}\xi_{n-1}^2\geqslant\\&\frac{1}{2}\xi_{n-1}^2+\frac{1}{2\beta_{n-1}^2(\cdot)+1}x_n^2,\end{aligned}\tag{13}$$

由此可得

$$\xi_1^2+\xi_2^2+\cdots+\xi_n^2\geqslant L_0(x_1^2+x_2^2+\cdots+x_n^2),\tag{14}$$

其中, $L_0=\max\left(\frac{1}{2\beta_i^2(\cdot)+1}\right)$, $i=1,2,\cdots,n$.

结合(3), (14)和引理 2, 下面不等式成立

$$\dot{W}\leqslant -L_0\sum_{i=1}^{n}\xi_i^2+\tau_n(x_1)\xi_n(u-u^*).\tag{15}$$

令

$$V_0=W+V,$$

利用(12)和(15), 导出

$$\begin{aligned}\dot{V}_0=\dot{W}+\dot{V}\leqslant&-L_0\sum_{i=1}^{n}\xi_i^2+\tau_n(x_1)\xi_n(u-u^*)+\\&\frac{1}{2}\sum_{i=2}^{n}\Big[\sum_{j=1}^{i} a_j(x_1)+\tau_i(x_1)-2l_{i-1}\tau_{i-1}(x_1)+l_{i-1}^2\tau_{i-1}(x_1)\sum_{j=2}^{i-1} l_j l_{j-1}\cdots l_2+\\&l_{i-1}\sum_{k=1}^{i-1} a_k(x_1)\sum_{j=2}^{i-1} c l_j l_{j-1}\cdots l_2\Big]e_i^2+\frac{1}{2}\sum_{i=2}^{n}\Big[l_{i-1}^2\tau_{i-1}(x_1)\sum_{j=2}^{i-1} l_j l_{j-1}\cdots l_2+\\&l_{i-1}\sum_{k=1}^{i-1} a_k(x_1)\sum_{j=2}^{i-1} c l_j l_{j-1}\cdots l_2\Big]e_j^2+\frac{1}{2}\sum_{i=2}^{n}\Big[\sum_{j=1}^{i} a_j(x_1)x_j^2+\tau_i(x_1)x_{i+1}^2\Big].\end{aligned}\tag{16}$$

取 $u(\hat{\boldsymbol{x}})=-b_n\xi_n(\hat{\boldsymbol{x}})$, $\xi_n(\hat{\boldsymbol{x}})=\hat{x}_n+b_{n-1}\hat{x}_{n-1}+\cdots+b_2\hat{x}_2+b_1x_1$, 结合(4), 得

$$\begin{aligned}|u-u^*|&=|-b_n(\xi_n-\xi_n^*)|=|-[b_n(x_n-\hat{x}_n)+b_{n-1}(x_{n-1}-\hat{x}_{n-1})+\cdots\\&+b_2(x_2-\hat{x}_2)+b_1(x_1-x_1)]|\leqslant b_n\left(\sum_{i=2}^{n-1} b_i\sum_{j=2}^{i} l_j l_{j-1}\cdots l_2|e_j|+\sum_{j=2}^{n} l_j l_{j-1}\cdots l_2|e_j|\right).\end{aligned}\tag{17}$$

利用引理 1, 也有

$$|\xi_n\tau_n(x_1)(u-u^*)|\leqslant M\tau_n(x_1)\xi_n^2+\tau_n(x_1)l_{n+1}(\xi_n(\boldsymbol{x})-\xi_n(\hat{\boldsymbol{x}}))^2,\tag{18}$$

其中, M, l_{n+1} 为正实数.

把(15), (17)和(18)代入(16), 有

$$\begin{aligned}\dot{V}_0\leqslant&-L_0\sum_{i=1}^{n}\xi_i^2+2M\tau_n(x_1)\sum_{i=1}^{n-1} b_i x_i^2+2M\tau_n(x_1)x_n^2+\\&2\tau_n(x_1)l_{n+1}\left(\sum_{i=2}^{n-1} b_i^2\sum_{j=2}^{i} l_j^2 l_{j-1}^2\cdots l_2^2|e_j|^2+\sum_{j=2}^{n} l_j l_{j-1}\cdots l_2|e_j|\right)+\end{aligned}$$

$$\frac{1}{2}\sum_{i=1}^{n}\Big[\sum_{j=1}^{i}a_j(x_1)+\tau_i(x_1)-2l_{i-1}\tau_{i-1}(x_1)+l_{i-1}^2\tau_{i-1}(x_1)\sum_{j=2}^{i-1}l_jl_{j-1}\cdots l_2+$$
$$l_{i-1}\sum_{k=1}^{i-1}a_k(x_1)\sum_{j=2}^{i-1}cl_jl_{j-1}\cdots l_2\Big]e_i^2+\frac{1}{2}\sum_{i=2}^{n}\Big[l_{i-1}^2\tau_{i-1}(x_1)\sum_{j=2}^{i-1}l_jl_{j-1}\cdots l_2+$$
$$l_{i-1}\sum_{k=1}^{i-1}a_k(x_1)\sum_{j=2}^{i-1}cl_jl_{j-1}\cdots l_2\Big]e_j^2+\frac{1}{2}\sum_{i=2}^{n}\Big[\sum_{j=1}^{i}a_j(x_1)x_j^2+\tau_i(x_1)x_{i+1}^2\Big]=$$
$$[-L_0+2M\tau_n(x_1)b_1]x_1^2+\sum_{i=2}^{n}\Big[-L_0+2M\tau_n(x_1)b_i+\frac{1}{2}\sum_{i=2}^{n-1}(a_i(x_1)+\tau_i(x_1))\Big]x_i^2+$$
$$\Big[-L_0+2M\tau_n(x_1)+\frac{1}{2}a_n(x_1)\Big]x_n^2+\frac{1}{2}\sum_{i=2}^{n}\Big[\sum_{j=1}^{i}a_j(x_1)+\tau_i(x_1)-2l_{i-1}\tau_{i-1}(x_1)+$$
$$l_{i-1}^2\tau_{i-1}(x_1)\sum_{j=2}^{i-1}l_jl_{j-1}\cdots l_2+l_{i-1}\sum_{k=1}^{i-1}a_k(x_1)\sum_{i=2}^{i-1}cl_jl_{j-1}\cdots l_2+4\tau_n(x_1)l_{n+1}l_i^2l_{i-1}^2\cdots l_2^2+$$
$$l_{j-1}^2\tau_{j-1}(x_1)\sum_{j=i-1}^{n}l_il_{i-1}\cdots l_2+l_{j-1}\sum_{k=i-1}^{n}a_k(x_1)\sum_{j=i}^{n}cl_il_{i-1}\cdots l_2+4\tau_n(x_1)l_{n+1}b_j^2\sum_{j=i}^{n}l_i^2l_{i-1}^2\cdots l_2^2\Big]e_i^2. \tag{19}$$

令

$$c_1=-L_0+2M\tau_n(x_1)b_1,$$
$$c_i=-L_0+2M\tau_n(x_1)b_i+\frac{1}{2}\sum_{i=1}^{n-1}a_i(x_1)+\tau_i(x_1),$$
$$c_n=-L_0+2M\tau_n(x_1)+\frac{1}{2}a_n(x_1),$$
$$q_i=\frac{1}{2}\Big[\sum_{j=1}^{i}a_j(x_1)+\tau_i(x_1)-2l_{i-1}\tau_{i-1}(x_1)l_{i-1}^2\tau_{i-1}(x_1)\sum_{j=2}^{i-1}l_jl_{j-1}\cdots l_2$$
$$+l_{i-1}\sum_{k=1}^{i-1}a_k(x_1)\sum_{j=2}^{i-1}cl_jl_{j-1}\cdots l_2+4\tau_n(x_1)l_{n+1}l_i^2l_{i-1}^2\cdots l_2^2+l_{j-1}^2\tau_{j-1}(x_1)\sum_{j=i-1}^{n}l_il_{i-1}\cdots l_2+$$
$$l_{j-1}\sum_{k=i-1}^{n}a_k(x_1)\sum_{j=i}^{n}cl_il_{i-1}\cdots l_2+4\tau_n(x_1)l_{n+1}b_j^2\sum_{j=i}^{n}l_i^2l_{i-1}^2\cdots l_2^2\Big], \tag{20}$$

由此, 可选取适当常数 L_0, M, b_i, 使得

$$c_i>p_0>0,\ q_j>p_0>0,\ i=1,2,\cdots,n,\ j=2,3,\cdots n. \tag{21}$$

所以, (19) 式可变为

$$\dot{V}_0\leqslant-\sum_{i=1}^{n}p_0x_i^2-\sum_{i=2}^{n}p_0e_i^2, \tag{22}$$

至此, 定理得证.

2 数值例子

考虑下面的非线性系统

$$\dot{x}_1=\frac{e^{-2}+x_1^2e^{-x_1^2}}{5e^{-2}+3x_1^2e^{-x_1^2}}x_2+\frac{x_1\sin x_1^2}{3+x_1^2}, \tag{23}$$
$$\dot{x}_2=u,\qquad y=x_1,$$

根据本文方法设计如下观测器

$$\dot{\hat{z}}_2=-2\left(\frac{e^{-2}+x_1^2e^{-x_1^2}}{5e^{-2}+3x_1^2e^{-x_1^2}}\hat{x}_2+\frac{x_1\sin x_1^2}{3+x_1^2}\right), \tag{24}$$
$$\hat{x}_2=\hat{z}_2+2x_1,$$

选取控制器为

$$u=-\hat{x}_2-\frac{1}{5}x_1,$$

仿真结果如下 $((x_1(0), x_2(0), \hat{z}_2(0))=(0.1, 0.2, 0.1))$:

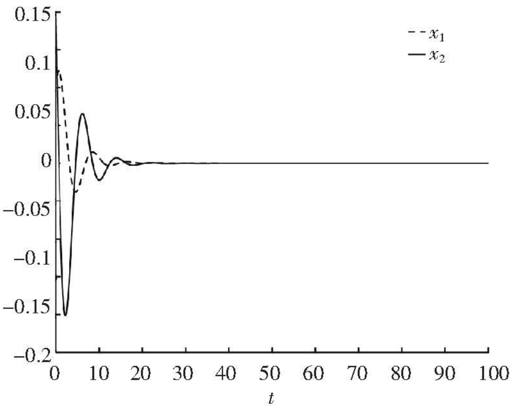

图 1　状态 x_1, x_2 的仿真

Fig.1　Simulation of state x_1 and x_2

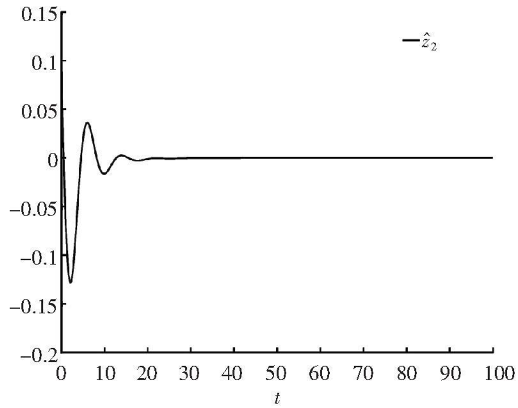

图 2　观测状态 $\hat{z}_2$ 的仿真

Fig.2　Simulation of observer state $\hat{z}_2$

3　结论

本文研究了一类非线性系统的全局渐近稳定问题. 利用观测设计方法, 建立了一个观测器和线性的输出补偿器, 得到了使系统全局渐近稳定的控制器.

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