Modal Identification Method of the Vibratory System Based on the Jump-Diffusion Process

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Abstract:In this paper,we put forward a new time-domain modal identification method of LTI system driven by a special Lévy process. The system response can be seen as a jump-diffusion process. Based on the properties of the quadratic variation and multi-power variation,the jump-diffusion process is decomposed into the diffusion process and the pure jump-driven process,both processes have the same unknown parameters as those included in the LTI system. The parameters of the LTI system are identified by the exact maximum likelihood estimation method of the diffusion process. The numerical results demonstrate that the method has high precision.

Key words:modal identification,jump-diffusion process,CAR model,multi-power variation,maximum likelihood estimator

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基于跳-扩散过程的振动系统的模态识别方法

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[摘要] 本文提出了 Lévy 激励下 LTI 系统的一种时域模态识别方法.系统响应可看做是一个跳-扩散过程.基于二次变差和多幂次变差的性质,跳-扩散过程被分解成扩散过程和纯跳激励的过程,二者都具有和原系统相同的未知参数.最后通过扩散过程的极大似然估计方法来估计 Lévy 激励下 LTI 系统的参数.数值结果表明该方法估计精度高.

[关键词] 模态识别,跳-扩散过程,CAR 模型,多幂变差,极大似然估计

Dynamical systems encountered in the physical world are usually of a continuous-time nature,therefore, identifying a continuous-time linear model using the continuous observations is appealing. With the development of sampling technology and computer technology,high frequency datum are relatively easy to collect,which makes the identification methods based on the continuous observations feasible.

In the ambient vibration test,the input is generally difficult to measure,so its distribution cannot be gotten easily. However,a lot of processes can be approximated by Lévy processes. Therefore,in the following discussion, we assume always the system is driven by a Lévy process. The structural dynamic equation is essentially a stochastic differential equation. For the LTI vibratory system,the stochastic differential equation can also be viewed as a continuous-time autoregressive model(CAR model). Therefore,modal identification of the LTI system can be implemented by estimating the physical parameters of the CAR model.

When driving processes are restricted to Brownian motions, many scholars discussed the parameter identification methods based on continuous observations,such as,DICS algorithm,the exact maximum likelihood

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estimation method based on the change of measure[1,2].

Under the Brownian motion excitation, the maximum likelihood estimation based on continuous observations is not only statistical excellent, but also easy to realize. Therefore, if the response under the Brownian motion excitation can be separated from the observed response, the modal identification of the system driven by the Lévy process can be implemented by the maximum likelihood estimation under the Brownian excitation. So, some scholars proposed separating methods by the multipower variation processes. In 2004, Barndorff-Nielsen for the first time introduced multipower variation processes, but only for finite jump cases[3]. In 2005, Woerner extended finite jumps to infinite jumps[4]. In 2009, GMM estimator based on the bipower and tripower variations was proposed by Todorov for the infinite jump cases[5]. In 2010, for many small jump case, Lee put forward the QQ chart method to detect small jumps[6]. Works above are mainly to detect jumps, the modal identification methods study still more less.

Herein, based on the modal identification method under the Brownian motion excitation, this paper presents an efficient time-domain identification procedure of LTI system driven by a special Lévy process to identify the physical parameters and the dynamic characteristics of a structural system by the multipower variation method.

1 The Structural Dynamic Equation and the Lévy-Driven CAR Model

The structural dynamic equation of a simple vibratory system excited by an unknown Lévy process can be expressed as follows,

$$M\ddot{x}(t)+C\dot{x}(t)+Kx(t)=\sigma(t)\mathrm{d}W(t)+\mathrm{d}\left(\sum_{i=1}^{N(t)}C_i\right)\ , \tag{1}$$

where M, C and K are mass, the vicious damping coefficient and stiffness respectively. $\ddot{x}(t)$, $\dot{x}(t)$ and $x(t)$ are the acceleration, velocity and displacement responses respectively, and $W(t)$ denotes the standard Brownian motion, $\sigma(t)$ is the diffusion coefficient, $\sum_{i=1}^{N(t)} C_i$ denotes the jump process, which is a compound Poisson process.

Assumed that mass of a system is known, the structural dynamic equation of a simple vibratory system excited by a Lévy process can be written as,

$$\ddot{x}(t)+2n\dot{x}(t)+p^2x(t)=\sigma(t)\mathrm{d}W(t)+\mathrm{d}\left(\sum_{i=1}^{N(t)}C_i\right)\ . \tag{2}$$

Eq. (2) can also be written as the observation and state equations

$$x(t)=(1,0)\boldsymbol{Y}(t), \tag{3}$$

$$\mathrm{d}\boldsymbol{Y}(t)=\boldsymbol{A}\boldsymbol{Y}(t)\mathrm{d}t+(0,1)^{\mathrm{T}}\left(\sigma(t)\mathrm{d}W(t)\ +\mathrm{d}\sum_{i=1}^{N(t)}C_i\right)\ , \tag{4}$$

where $\boldsymbol{Y}(t)=(y_0(t),y_1(t))^{\mathrm{T}}$.

Our goal is to identify the parameters based on the known Lévy-driven responses. However, when the system is excited by the special Lévy process— Brownian-jump process, the stochastic response is no longer Gaussian. Therefore, the maximum likelihood estimator via the theory of change of measure becomes maximum pseudolikelihood estimator, and the estimators are typically inefficient[7]. However, due to the linearity of the system, the stochastic response can be decomposed into the Brownian-driven component and the jump-driven response, and every term satisfies Eq. (4), which means the modal identification can be obtained by the methods when the system is excited by the Brownian motion.

The following questions are how to decompose the response into the Brownian-driven response and the jump-driven response, the theory of quadratic variation and multi-power variation provides a better thought.

2 Quadratic Variation and Multi-Power Variation Processes

2.1 Definition, properties and estimation

In this section, we recall some basic definitions and results needed for the study of the system driven by a

Lévy process. The quadratic variation, bipower variation and quadpower variation processes of the jump-diffusion process $y(t)$ are defined respectively, when they exist, as[3]

$$[y]^{[2]}(t)=L^2-\lim_{n\to\infty}\sum_{j=1}^{n}|y(t_{j+1}^{(n)})-y(t_j^{(n)})|^2,$$

$$[y]^{[1,1]}(t)=L^2-\lim_{n\to\infty}\sum_{j=1}^{n}|y(t_{j+2}^{(n)})-y(t_{j+1}^{(n)})||y(t_{j+1}^{(n)})-y(t_j^{n)})|,$$

$$[y]^{[1,1,1,1]}(t)=L^2-\lim_{n\to\infty}\sum_{j=1}^{n}|y(t_{j+4}^{(n)})-y(t_{j+3}^{(n)})||y(t_{j+3}^{(n)})-y(t_{j+2}^{(n)})|\times$$
$$|y(t_{j+2}^{(n)})-y(t_{j+1}^{(n)})||y(t_{j+1}^{(n)})-y(t_j^{(n)})|,$$

where L^2 denotes the convergence in the mean square meaning. The properties of above variation processes are as follows,

$$[y]^{[2]}(t)=\int_0^t\sigma^2(s)\,\mathrm{d}s+\sum_{i=1}^{N(t)}C_i^2,$$

$$[y]^{[1,1]}(t)=\frac{2}{\pi}\int_0^t\sigma^2(s)\,\mathrm{d}s,$$

$$[y]^{[1,1,1,1]}(t)=\left(\frac{2}{\pi}\right)^2\int_0^t\sigma^4(s)\,\mathrm{d}s.$$

In the continuous record case, we can easily estimate the parameters controlling $\sigma^2(t)$ and the jumps by making direct inference on quadratic variation and multi-power variation. In practice, we observe the datum only at discrete times and therefore such estimation is infeasible. However, if we have high-frequency observations of the response, we can make the estimation feasible by substituting quadratic variation and multi-power variation with realized measures. The estimators of the quadratic variation, bipower variation and quadpower variation are constructed as the following, respectively,

$$\widehat{[y]^{[2]}(t)}=\sum_{j=1}^{[t/\delta]-1}|y((j+1)\delta)-y(j\delta)|^2,$$

$$\widehat{[y]^{[1,1]}(t)}=\sum_{j=1}^{[t/\delta]-2}|y((j+2)\delta)-y((j+1)\delta)||y((j+1)\delta)-y(j\delta)|,$$

$$\widehat{[y]^{[1,1,1,1]}(t)}=\sum_{j=1}^{[t/\delta]-4}|y((j+4)\delta)-y((j+3)\delta)||y((j+3)\delta)-y((j+2)\delta)|\times$$
$$|y((j+2)\delta)-y((j+1)\delta)||y((j+1)\delta)-y(j)\delta)|.$$

When $\delta\to0$, we have

$$\widehat{[y]^{[2]}(t)}\xrightarrow{P}\int_0^t\sigma^2(s)\,\mathrm{d}s+\sum_{i=1}^{N(t)}C_i^2,$$

$$\widehat{[y]^{[1,1]}(t)}\xrightarrow{P}\frac{2}{\pi}\int_0^t\sigma^2(s)\,\mathrm{d}s,$$

$$\widehat{[y]^{[1,1,1,1]}(t)}\xrightarrow{P}\left(\frac{2}{\pi}\right)^2\int_0^t\sigma^4(s)\,\mathrm{d}s.$$

Obviously,

$$\widehat{[y]^{[2]}(t)}-\frac{\pi}{2}\widehat{[y]^{[1,1]}(t)}\xrightarrow{P}\sum_{i=1}^{N(t)}C_i^2,\tag{5}$$

also under the hypothesis that the interval $[0,t]$ doesn't exist jump,

$$Z=\frac{\widehat{[y]^{[2]}(t)}-\frac{\pi}{2}\widehat{[y]^{[1,1]}(t)}}{\sqrt{\left(\frac{\pi}{2}\right)^2\widehat{[y]^{[1,1,1,1]}(t)}}}\xrightarrow{D}N(0,0.609\ 1).\tag{6}$$

2.2 Separating methods

For fulfilling separation of the jump-driven response and the Brownian-driven response, we first need to identify the jump times and sizes from the Lévy-driven response. Since in a long time interval, the number of jumps can be more than one, which means the estimator in Eq. (5) is the square sum of many jumps, so we can't identify each jump time and size through Eq. (5). Therefore, we consider to divide the long time interval into consecutive subintervals, and each subinterval is assumed to involve only one jump.

The setting is as following. Starting from the left endpoint of each subinterval, then gradually adding the data points, and computing $\widehat{[y]^{[2]}(t)-\frac{\pi}{2}[y]^{[1,1]}(t)}$. When the jump time arrives, $\widehat{[y]^{[2]}(t)-\frac{\pi}{2}[y]^{[1,1]}(t)}$ will change suddenly, so, we think there exists one jump at the time, and the square root of $\widehat{[y]^{[2]}(t)-\frac{\pi}{2}[y]^{[1,1]}(t)}$ is the jump size.

Of course, when the jump size is smaller, the change can't often be identified accurately, so we can use the asymptotic normality property in Eq. (6). When $|z|<3\sqrt{0.6091}$, we think no jump in the subinterval, otherwise there exists one jump.

In some cases, the jump time may be at the left endpoint. For this case, we consider to translate the subintervals to construct the new divisions of the long time interval, $[t_1-\delta, t_2-\delta), \cdots, [t_{k-1}-\delta, t_k-\delta)$ or $[t_0+\delta, t_1+\delta), \cdots, [t_{k-2}+\delta, t_{k-1}+\delta)$. With the vary of δ, we can obtain many divisions. Under each division, we get estimators of the time and the size of jumps. Merging the results can obtain the estimators of the jump time and size, then the jump-driven response can be obtained by using the maximum pseudolikelihood estimators computed from the initial response as the initial estimators. Finally, we can get the Brownian-driven response.

2.3 Maximum pseudolikelihood estimation

The maximum pseudolikelihood estimation method of a Lévy-driven system was presented by Du in 2010.

Let $\tilde{\boldsymbol{Y}}(t)=(y_1(t), y_0(t))^{\mathrm{T}}$, where $y_i(t)$ $(i=0,1)$ is defined in Eq. (4). The exact maximum pseudolikelihood estimator of parameters is given by

$$\widehat{(2n, p^2)}=-\int_0^T \frac{\tilde{\boldsymbol{Y}}^{\mathrm{T}}(t)}{\sigma^2(t)}\mathrm{d}y_1(t)\left(\int_0^T \frac{\tilde{\boldsymbol{Y}}(t)\tilde{\boldsymbol{Y}}^{\mathrm{T}}(t)}{\sigma^2(t)}\mathrm{d}t\right)^{-1}. \tag{7}$$

2.4 Maximum likelihood estimation

The maximum likelihood estimation method of a diffusion process was presented by Brockwell in 2007, parameter $\sigma(t)$ could be estimated by the quadratic variation.

If the separating Brownian-driven response is denoted as $\boldsymbol{Z}(t)=(z_0(t), z_1(t))^{\mathrm{T}}$, then we have

$$\mathrm{d}\boldsymbol{Z}(t)=A\boldsymbol{Z}(t)\mathrm{d}t+(0,1)^{\mathrm{T}}\sigma(t)\mathrm{d}W(t).$$

Let $\tilde{\boldsymbol{Z}}(t)=(z_1(t), z_0(t))^{\mathrm{T}}$, then the exact maximum likelihood estimator of parameters is given by

$$\widehat{(2n, p^2)}=-\int_0^T \frac{\tilde{\boldsymbol{Z}}^{\mathrm{T}}(t)}{\sigma^2(t)}\mathrm{d}z_1(t)\left(\int_0^T \frac{\tilde{\boldsymbol{Z}}(t)\tilde{\boldsymbol{Z}}^{\mathrm{T}}(t)}{\sigma^2(t)}\mathrm{d}t\right)^{-1}. \tag{8}$$

3 Simulations

To demonstrate the effectiveness and robustness of the system identification methodology presented above, a simple vibration system with the following properties is considered. The mass is 549 693 kg, the viscous damping coefficient and stiffness are 554 000 Ns/m and 365 800 000 N/m respectively, natural frequency is 4.107 9 Hz, modal damping ratio is 0.019 5.

Assume the structure is excited by the Brownian-jump process, furthermore assume the jump process is a compound Poisson process. The sampling interval is 0.001 s, and the displacement response data in 5 minutes is obtained by simulation.

Using the method in section 2, we identify the time and size of jumps from the response. The theoretic values and the identified results are presented in Table 1. We can see from it that for large jumps, the methods are very valid, the jump times can be identified accurately, and the sizes are also close to the true values. But some small jumps can't be identified. We also see that the identified procedure may bring some false small jumps, which will influence the identification accuracy.

Table 1 Jump times and sizes

Theoretic values		Identified results		Theoretic values		Identified results	
Time/s	Size	Time/s	Size	Time/s	Size	Time/s	Size
		2.881	0.256 3	133.676	−4.476 4	133.676	−3.928 8
		6.456	0.645 1	146.930	2.820 9	146.930	2.797
10.334	−1.594 6	10.334	−1.568 3	149.510	−10.748 6	149.510	−10.681
		17.331	−0.270 1			157.406	−0.641 8
		34.084	−0.461 8	175.581	22.155 2	175.581	22.125 6
35.700	1.390 6	35.700	1.257 1	193.894	−0.403 5		
		40.505	−0.463 4	196.530	9.095 2	196.530	8.891 1
48.784	−2.948 3	48.784	−2.909	203.911	1.151 6	203.911	0.887 6
		49.204	0.264 2	208.449	5.836 4	208.449	5.799 8
56.062	−0.181 3			212.810	−2.695 1	212.810	−2.420 5
67.144	−3.762 8	67.144	−3.741 5			218.608	0.677 3
82.808	−0.382 8	82.808	−0.309 9	226.407	8.989 4	226.407	8.982 7
95.130	−1.238 5	95.130	−1.042 5	229.656	2.537 6	229.656	1.616 4
102.116	3.166 9	102.116	2.458 5	238.056	−0.187 2		
114.468	1.247 6	114.468	1.248 3	247.038	−3.409 2	247.038	−3.148 5
		123.006	0.271 9			258.105	−0.349 1
		130.330	0.452 1	285.067	−2.665 1	285.067	−2.631 8
131.624	0.341 3			287.924	−3.854 6	287.924	−3.628 9

The identified damping coefficients are presented in column (2) of Table 2, relative errors are shown in column (3). Column (1) denote the mean jump numbers in 5 min, which are 39, 81, 123, 165, 207, respectively. As the number of jumps increase, the relative error is increasing gradually. But for the stiffness (see column (4) and (5) of Table 2), these identified results are very close to the theoretical values. For all cases, the maximum error in identifying the stiffness is less than 0.3%.

Table 2 Identified damping coefficients and stiffnesses

Number of jumps	Damping results		Stiffness results	
	Damping/(10^5 Ns/m)	Relative error	Stiffness/(10^8 N/m)	Relative error
39	5.664 9	0.022 5	3.652 4	0.002 6
81	5.713 7	0.031 3	3.656 7	0.001 4
123	5.773 5	0.042 1	3.653 8	0.002 2
165	5.926 4	0.069 7	3.654 1	0.002 2
207	6.246 9	0.127 6	3.654 0	0.002 2

We calculate also modal parameters by eigen-analysis after obtaining the physical parameters of the system. The identified results and errors of the natural frequency and the damping ratio are presented in Table 3. It is observed from Table 3 that these identified results are very close to the theoretical values. For all cases, the maximum error in identifying the natural frequencies is less than 0.15%, whereas the maximum error in identifying the damping ratio is relative large with the increase of the number of jumps.

Table 3 Identified natural frequencies and damping ratios

Number of jumps	Frequency results		Damping ratio results	
	Frequency/Hz	Relative error	Damping ratio	Relative error
39	4.102 5	0.001 3	0.020 0	0.023 9
81	4.104 9	0.000 7	0.020 2	0.032 1
123	4.103 3	0.001 1	0.020 4	0.043 3
165	4.103 4	0.001 1	0.020 9	0.070 9
207	4.103 4	0.001 1	0.022 0	0.128 8

4 Conclusions

The paper presents a new method for identifying modal parameters of the structure driven by the Brownian-jump process based on the multi-power variations. It is clear from the above analysis that the new modal identification method has very high computing efficiency and accuracy in identifying stiffness and natural frequency, but in identifying damping and damping ratio, the error is relative large. However, compared to the maximum pseudolikelihood estimation, the estimation accuracy has increased.

[参考文献]

[1] Brockwell P, Davis R A, Yu Y, et al. Continuous-time Gaussian Autoregression[J]. Statistica Sinica, 2007, 17(1): 63-80.

[2] Du X L, Wang F Q. Modal identification based on Gaussian continuous time autoregressive moving average model[J]. Journal of Sound and Vibration, 2010, 329(20): 4 294-4 312.

[3] Barndorff-Nielsen O E, Shephard N. Power and bipower variation with stochastic volatility and jumps[J]. Journal of Financial Econometrics, 2004, 2(1): 1-37.

[4] Woerner H C J. Estimation of integrated volatility in stochastic volatility models[J]. Applied Stochastic Models in Business and Industry, 2005, 21(1): 27-44.

[5] Todorov V. Estimation of continuous-time stochastic volatility models with jumps using high-frequency dada[J]. Journal of E-conometrics, 2009, 148(2): 131-148.

[6] Lee S, Hannig J. Detecting jumps from Lévy jump diffusion processes[J]. Journal of Financial Economics, 2010, 96(2): 271-290.

[7] Fan J Q, Yao Q W. Nonlinear Time Series: Nonparametric and Parametric Methods[M]. New York: Springer-Verlag, 2005.

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