A Projective Dynamic Method for Solving Linear Programming

Sun Liming

(School of Science,Nanjing Audit University,Nanjing 211815,China)

Abstract: In this paper, we propose a projective dynamic method for minimizing general linear programming. The new method is based on the variational inequality(VI)properties. We extend the variational inequality method to construct a new ODE system. The preliminary numerical results are reported and the new dynamics is shown to be very useful to solve large scale optimization problems.

Key words: linear programming, continuous method, variational inequality, projective dynamic method

CLC number: 90C25, 68Q25, 62J05 **Document code**: A **Article ID**: 1001-4616(2015)04-0008-06

一种求解线性规划的投影动态方法

孙黎明

(南京审计学院理学院,江苏 南京 211815)

[摘要] 提出了一种求解线性规划问题的投影动态方法.新方法是基于变分不等式的理论和性质而提出的.将变分不等式的方法进行推导并构建了一个新的ODE系统.论文给出了初步的试验结果,表明了算法的有效性.新方法将用于解决大规模的优化问题.

[关键词] 线性规划,连续性方法,变分不等式,投影动态方法

Consider a linear programming problem in canonical form as follows

$$
\begin{aligned}
(P)\quad \min\quad & \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x} \\
s.t.\quad & \boldsymbol{Ax} \leqslant \boldsymbol{b}, \\
& \boldsymbol{x}_i > 0,\ i = 1,2,\cdots,n.
\end{aligned}
\tag{1}
$$

where $\boldsymbol{A}$ is an $m \times n$ matrix, $\operatorname{rank}(\boldsymbol{A}) = m$ and $\boldsymbol{b}, \boldsymbol{c}$ are vectors of length m and n respectively. For problem(1),we define the feasible set as

$$\Omega_0 = \left\{ \boldsymbol{x} \in R^n \middle| \boldsymbol{Ax} \leqslant \boldsymbol{b},\ \boldsymbol{x} \geqslant 0 \right\}$$

and the optimal solution set as $\Omega_0^* = \{\boldsymbol{x} \in \boldsymbol{R}^n | \boldsymbol{x}$ is an optimal solution of (1)$\}$.

First,let us state some assumptions on the problem that we are interested in.

Assumption 1 (a) There exists relative-interior feasible points for problem(1).

(b) The set of optimal solutions of problem(1) is nonempty and bounded.

(c) The matrix $\boldsymbol{A}$ is full row rank and c is not in the range space of $\boldsymbol{A}^{\mathrm{T}}$.

The above assumptions are standard in the literature.

The KKT condition for problem(1) can be written as follows

$$
\begin{cases}
\boldsymbol{x}^{\mathrm{T}}(\boldsymbol{c} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{y}) = 0, & \boldsymbol{x} \geqslant 0,\ \boldsymbol{c} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{y} \geqslant 0, \\
\boldsymbol{y}^{\mathrm{T}}(-\boldsymbol{Ax} + \boldsymbol{b}) = 0, & \boldsymbol{y} \geqslant 0,\ \boldsymbol{Ax} \leqslant \boldsymbol{b}.
\end{cases}
\tag{2}
$$

Received data: 2015-04-17.

Foundation item: Supported by the National Natural Science Foundation of China(71302178,11301271)

Corresponding author: Sun Liming, doctor, majored in numerical optimization. E-mail: sunli_ming@163.com

From Theorem 9.4.2 in Ref.[1], we know that $x \in \Omega_0^*$ is equivalent to finding x, y satisfying (2).

In the next section, we want to convert the linear programming into a variational inequality problem. We propose a continuous method based on variational method. The convergence of our method is given for any starting point.

1 Equivalent Variational Inequality Problem

Let us define

$$\boldsymbol{u} = \begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{pmatrix},\ \boldsymbol{F}(\boldsymbol{u}) = \begin{pmatrix} \boldsymbol{c} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{y} \\ -\boldsymbol{A}\boldsymbol{x} + \boldsymbol{b} \end{pmatrix} = \boldsymbol{M}\boldsymbol{u} + \boldsymbol{q}\ , \tag{3}$$

where

$$\boldsymbol{M} = \begin{pmatrix} \boldsymbol{0} & \boldsymbol{A}^{\mathrm{T}} \\ -\boldsymbol{A} & \boldsymbol{0} \end{pmatrix},\ \boldsymbol{q} = \begin{pmatrix} \boldsymbol{c} \\ \boldsymbol{b} \end{pmatrix}.$$

and the following linear variational inequality problem

$$(VI(\Omega, \boldsymbol{M}, \boldsymbol{q}))\ \text{find}\ a\ \boldsymbol{u}^* \in \Omega\ \text{such that}\ (\boldsymbol{u} - \boldsymbol{u}^*)^{\mathrm{T}}\boldsymbol{F}(\boldsymbol{u}) \geqslant 0,\ \forall \boldsymbol{u} \in \Omega, \tag{4}$$

where

$$\Omega = \left\{ \boldsymbol{u} = \begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{pmatrix} \middle| \boldsymbol{x} \in R_+^{\ n},\ \boldsymbol{y} \in R_+^m \right\}.$$

Lemma 1 $\boldsymbol{F}(\boldsymbol{u})$ in (3) is monotone on Ω.

Proof It is similar to the proof of Lemma 2 in Liao's paper[2].

Lemma 1 shows that (4) is a monotone variational inequality problem. Let Ω^* be the optimal solution set of (4). Then, we get another lemma.

Lemma 2 u satisfies (2) if and only if $\boldsymbol{u} \in \Omega^*$.

Proof Let $\boldsymbol{u} = (\boldsymbol{x}^{\mathrm{T}}, \boldsymbol{y}^{\mathrm{T}})^{\mathrm{T}}$.

($\Rightarrow$) Since $\boldsymbol{u}$ satisfies (2), then $\boldsymbol{u} \in \Omega$ and $\boldsymbol{u}' = (\boldsymbol{x}^{\mathrm{T}}, \boldsymbol{y}^{\mathrm{T}})^{\mathrm{T}} \in \Omega$, we have

$$(\boldsymbol{u}' - \boldsymbol{u})^{\mathrm{T}}\boldsymbol{F}(\boldsymbol{u}) = \boldsymbol{u}'^{\mathrm{T}}\boldsymbol{F}(\boldsymbol{u}) - \boldsymbol{u}^{\mathrm{T}}\boldsymbol{F}(\boldsymbol{u}) = \boldsymbol{u}'^{\mathrm{T}}\boldsymbol{F}(\boldsymbol{u}) \tag{5}$$

From $\boldsymbol{u}' \geqslant 0, \boldsymbol{F}(\boldsymbol{u}) \geqslant 0$ and (5), we get that

$$(\boldsymbol{u}' - \boldsymbol{u})^{\mathrm{T}}\boldsymbol{F}(\boldsymbol{u}) \geqslant 0,\quad \forall \boldsymbol{u}' \in \Omega.$$

The proof of the necessary part is completed.

($\Leftarrow$) Since $\boldsymbol{u} \in \Omega^*$, then $\boldsymbol{u} \geqslant 0$. Let us set $\boldsymbol{u}' = \boldsymbol{0}$ and $\boldsymbol{u}' = 2\boldsymbol{u}$ respectively, substitution into (4), we obtain that

$$\boldsymbol{u}^{\mathrm{T}}\boldsymbol{F}(\boldsymbol{u}) \leqslant 0 \text{ and } \boldsymbol{u}^{\mathrm{T}}\boldsymbol{F}(\boldsymbol{u}) \geqslant 0.$$

This implies that $\boldsymbol{u}^{\mathrm{T}}\boldsymbol{F}(\boldsymbol{u}) = 0$.

Next, we prove that $\boldsymbol{F}(\boldsymbol{u}) \geqslant 0$ by contradiction.

Suppose that there exists an $i \in \{1, 2, \cdots, n\}$ such that $\boldsymbol{F}_i(\boldsymbol{u}) < 0$. Let $\boldsymbol{u}' = \boldsymbol{u} + \boldsymbol{e}_i$, where $\boldsymbol{e}_i$ is the ith column vector of the identity matrix $\boldsymbol{I}_{m+n}$. Then, we have

$$(\boldsymbol{u}' - \boldsymbol{u})^{\mathrm{T}}\boldsymbol{F}(\boldsymbol{u}) = \boldsymbol{e}_i^{\mathrm{T}}\boldsymbol{F}(\boldsymbol{u}) = \boldsymbol{F}_i(\boldsymbol{u}) < 0.$$

This is a contradiction with $\boldsymbol{u} \in \Omega^*$. Therefore, $\boldsymbol{u}$ satisfies (2). This proves Lemma 2.

The above lemmas show that finding an $\boldsymbol{x} \in \Omega_0^*$ is equivalent to finding $\boldsymbol{u} \in \Omega^*$, solving a monotone variational inequality problem.

2 Some Preliminaries of Projection and Variational Inequalities

The concept of the projection mapping is very fundamental in this paper. Without loss of generality, we regard the dimension of Ω as n. The solution of problem

$$\min\{\|\boldsymbol{x} - \boldsymbol{y}\| \,|\, \boldsymbol{x} \in \Omega\}$$

is called the projection of y onto Ω , denoted by $P_\Omega(y)$. This can be also written as

$$P_\Omega(y)=\arg\min\{\|x-y\| \| x\in\Omega\}.$$

A basic property of the projection mapping on a closed convex set is that

$$(y-P_\Omega(y))^T(x-P_\Omega(y))\leqslant 0,\quad \forall y\in R^n, \forall x\in\Omega. \tag{6}$$

Fig.1 gives its geometric interpretation.

We show some basic properties of the projection mapping in the following Lemma.

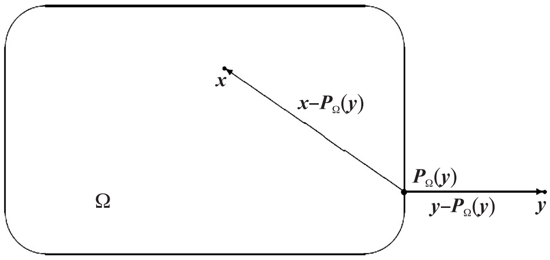

Fig.1　Geometric implementation of inequality(6)

Lemma 3　Let $\Omega\subset R^n$ be a convex closed set, then we have

$$\|P_\Omega(x)-P_\Omega(x)\|\leqslant\|x-y\|,\quad \forall x,y\in R^n. \tag{7}$$

$$\|P_\Omega(y)-x\|\leqslant\|x-y\|,\quad \forall y\in R^n, x\in\Omega. \tag{8}$$

$$\|P_\Omega(y)-x\|^2\leqslant\|x-y\|^2-\|y-P_\Omega(y)\|^2,\quad \forall y\in R^n, x\in\Omega. \tag{9}$$

Some other properties about the $VI(\Omega, M, q)$ are presented in [3]. From the early work of Eaves [4], we know that solving a variational inequality $VI(\Omega, M, q)$ is equivalent to the following projection equation

$$u=P_\Omega[u-(Mu+q)]. \tag{10}$$

In other words, to solve $VI(\Omega, M, F)$ is equivalent to finding a zero point of the residual function

$$e(u):=u-P_\Omega[u-F(u)],\ F(u)=Mu+q. \tag{11}$$

Theorem 1[4,5]　Let Ω be a nonempty closed convex subset of R^n and $\beta>0$. Then u^* is a solution of $VI(\Omega, M, q)$ if and only if $e(u^*,\beta)=0$, where $e(u^*,\beta):=u^*-P_\Omega[u^*-\beta F(u^*)]$, $F(u^*)=Mu^*+q$.

From Theorem 1 and the continuity of $e(u)$, we see that $\|e(u)\|$ can measure how much u fails to be in Ω^* .

Let $u^*\in\Omega^*$ be a solution. For any $u\in R^n$, $P_\Omega[u-F(u)]\in\Omega$. It follows from (4) that

$$(F1)\quad F(u^*)^{\mathrm T}\{P_\Omega[u-F(u)]-u^*\}\geqslant 0,\ \forall u\in R^n. \tag{12}$$

Setting $y=u-F(u)$ and $x=u^*$ in (6), we get that

$$(F2)\quad \{e(u)-F(u)\}^{\mathrm T}\{P_\Omega[u-F(u)]-u^*\}\geqslant 0,\ \forall u\in R^n. \tag{13}$$

Adding (12) and (13), we have

$$\{e(u)-(F(u)-F(u^*))\}^{\mathrm T}\{(u-u^*)-e(u)\}\geqslant 0,\ \forall u\in R^n. \tag{14}$$

For $F(u)=Mu+q$, M is positive semidefinite, we have the following theorem.

Theorem 2　For all $u^*\in\Omega^*$ and $u\in R^n$, so we get

$$(u-u^*)^{\mathrm T}\mathrm d(u)\geqslant\|e(u)\|^2,\quad \forall u\in R^n, \tag{15}$$

where

$$\mathrm d(u)=(I+M^{\mathrm T})e(u) \tag{16}$$

In the next section, we use this search direction $\mathrm d(u)$ and a merit function to propose a projective dynamics.

1.4　Projective Dynamic Method

Now, we propose our continuous method for solving the problem (4), which consists of a merit function and a ODE system. In our continuous method, the merit function is

$$E(u)=\frac{1}{2}\|u-u^*\|^2. \tag{17}$$

The projective dynamics

$$\frac{\mathrm du(t)}{\mathrm dt}=-\alpha(u)\mathrm d(u), \tag{18}$$

where

$$\mathrm{d}(\boldsymbol{u}) = (\boldsymbol{I} + \boldsymbol{M}^{\mathrm{T}})\boldsymbol{e}(\boldsymbol{u}),\ \ \alpha(\boldsymbol{u}) = \frac{\left\|\boldsymbol{e}(\boldsymbol{u})\right\|^2}{\left\|(\boldsymbol{I} + \boldsymbol{M}^{\mathrm{T}})\boldsymbol{e}(\boldsymbol{u})\right\|^2}. \tag{19}$$

1.5 Convergence Analysis of Our Continuous Method

In this section, we will give the convergence properties for (18).

Theorem 3 For any $\boldsymbol{u}_0 \in R^{m+n}$, there is a solution $\boldsymbol{u}(t)$ of (18), with $\boldsymbol{u}(0) = \boldsymbol{u}_0$ and $\boldsymbol{u}(t)$ defined in $[0, \infty]$.

Proof It follows from Assumption 1 that there is a finite $\boldsymbol{x}^* \in \Omega_0^*$, so there is a finite $\boldsymbol{u}^* \in \Omega^*$, then

$$\frac{\mathrm{d}E(\boldsymbol{u})}{\mathrm{d}t} = \frac{1}{2}\frac{\mathrm{d}\left\|\boldsymbol{u} - \boldsymbol{u}^*\right\|^2}{\mathrm{d}t} = -\alpha(\boldsymbol{u})(\boldsymbol{u} - \boldsymbol{u}^*)^{\mathrm{T}}\mathrm{d}(\boldsymbol{u}), \tag{20}$$

by using Theorem 2, we get that

$$\frac{\mathrm{d}\left\|\boldsymbol{u} - \boldsymbol{u}^*\right\|^2}{\mathrm{d}t} = -2\alpha(\boldsymbol{u})(\boldsymbol{u} - \boldsymbol{u}^*)^{\mathrm{T}}\mathrm{d}(\boldsymbol{u}) \leqslant -2\alpha(\boldsymbol{u})\left\|\boldsymbol{e}(\boldsymbol{u})\right\|^2 \leqslant 0. \tag{21}$$

(21) shows that $\boldsymbol{u}(t) \in B(\boldsymbol{u}_0, \boldsymbol{u}^*) := \left\{\boldsymbol{u} \in R^{n+m} \,\middle|\, \left\|\boldsymbol{u} - \boldsymbol{u}^*\right\| \leqslant \left\|\boldsymbol{u}_0 - \boldsymbol{u}^*\right\|\right\}$. Since the set $B(\boldsymbol{u}_0, \boldsymbol{u}^*)$ is a closed bounded set, then the right-hand side function of (18) is bounded and continuous. The result is obtained by using the Cauchy-Peano theorem.

Next, let us prove the convergence result.

Theorem 4 For any $\boldsymbol{u}_0 \in R^{m+n}$, let $\boldsymbol{u}(t)$ be a solution of (18) with $\boldsymbol{u}(0) = \boldsymbol{u}_0$. Then, $\lim\limits_{t \to +\infty} \boldsymbol{u}(t)$ exists and $\lim\limits_{t \to +\infty} \boldsymbol{u}(t) = \bar{\boldsymbol{u}} \in \Omega^*$.

Proof From Theorem 2, we have that

$$\frac{\mathrm{d}E(\boldsymbol{u})}{\mathrm{d}t} = \frac{1}{2}\frac{\mathrm{d}\left\|\boldsymbol{u} - \boldsymbol{u}^*\right\|^2}{\mathrm{d}t} = -\alpha(\boldsymbol{u})(\boldsymbol{u} - \boldsymbol{u}^*)^{\mathrm{T}}\mathrm{d}(\boldsymbol{u}) \leqslant 0, \quad \forall t \geqslant 0.$$

From the LaSalle invariant-set theorem, we have that

$$\lim_{t \to +\infty} \boldsymbol{e}(\boldsymbol{u}(t)) = \boldsymbol{0}. \tag{22}$$

In the proof of Theorem 3, we get that $\boldsymbol{u}(t) \in B(\boldsymbol{u}_0, \boldsymbol{u}^*)$, which is a compact set.

Therefore, there is a sequence $t_k \to +\infty$ as $k \to +\infty$, so $\boldsymbol{u}(t_k)$ has a limit $\bar{\boldsymbol{u}}$, denoted as

$$\lim_{k \to \infty} \boldsymbol{u}(t_k) = \bar{\boldsymbol{u}}.$$

Using formula (22), we get

$$\boldsymbol{e}(\bar{\boldsymbol{u}}) = \boldsymbol{0}.$$

From Theorem 1, we have that

$$\boldsymbol{e}(\bar{\boldsymbol{u}}) = \boldsymbol{0} \Leftrightarrow \bar{\boldsymbol{u}} \in \Omega^*.$$

By replacing $\boldsymbol{u}^*$ by $\bar{\boldsymbol{u}}$ in (21), we get that

$$\frac{\mathrm{d}\left\|\boldsymbol{u} - \bar{\boldsymbol{u}}\right\|^2}{\mathrm{d}t} = -2\alpha(\boldsymbol{u})(\boldsymbol{u} - \bar{\boldsymbol{u}})^{\mathrm{T}}\mathrm{d}(\boldsymbol{u}) \leqslant -2\alpha(\boldsymbol{u})\left\|\boldsymbol{e}(\boldsymbol{u})\right\|^2 \leqslant 0.$$

This formula and $\lim\limits_{k \to +\infty} \boldsymbol{u}(t) = \bar{\boldsymbol{u}}$ imply

$$\lim_{t \to +\infty} \boldsymbol{u}(t) = \bar{\boldsymbol{u}}.$$

The proof of the theorem is completed.

Remark 1 We present a new continuous method based on variational inequalities for linear programming problems. The complete convergence results of our continuous method are obtained. Numerical result given in section 2 demonstrates that our method is effective.

3 Numerical Experiments

In this part, we present some numerical results of our projective dynamic method. All our experiments are carried out on a computer with a Dell Pentium (R) CPU 3.40GHz and 2GB RAM on the MATLAB (2007b) platform. We give some small examples to verify the efficiency of our methods and show the trajectories of our method approaching optimal solutions. The problems are described as the following linear programming problems

Example 1

$$\begin{aligned} \min \quad & -4x_1-3x_2 \\ s.t. \quad & x_1 + x_2 + x_3 = 40, \\ & 2x_1 + x_2 + x_4 = 60, \\ & x_i \geqslant 0, \quad i=1,2,3,4. \end{aligned}$$

Let us consider the equivalent linear program as follows

$$\begin{aligned} \min \quad & -4x_1-3x_2 \\ s.t. \quad & x_1+x_2 \leqslant 40, \\ & 2x_1+x_2 \leqslant 60, \\ & x_i \geqslant 0, \ i=1,2. \end{aligned}$$

Let

$$\boldsymbol{A}=\begin{pmatrix}1 & 1\\ 2 & 1\end{pmatrix},\ \boldsymbol{b}=\begin{pmatrix}40\\60\end{pmatrix} \text{ and } \boldsymbol{c}=\begin{pmatrix}-4\\-3\end{pmatrix}.$$

The linear programming problem is equivalent to a linear variational inequality problem. We use the denotations as follows

$$\boldsymbol{u}=\begin{pmatrix}\boldsymbol{x}\\ \boldsymbol{y}\end{pmatrix}, \boldsymbol{M}=\begin{pmatrix}0 & \boldsymbol{A}^T\\ -\boldsymbol{A} & 0\end{pmatrix}, \boldsymbol{q}=\begin{pmatrix}\boldsymbol{c}\\ \boldsymbol{b}\end{pmatrix},$$

and

$$\Omega=\left\{\boldsymbol{u}=\begin{pmatrix}\boldsymbol{x}\\ \boldsymbol{y}\end{pmatrix}\middle|\, \boldsymbol{x}\in R_+^2,\ \boldsymbol{y}\in R_+^2\right\}.$$

The optimal solution of this problem is

$$\boldsymbol{u}^*=(20,\ 20,\ 2,\ 1)^{\mathrm{T}},\ \boldsymbol{x}^*=(20,\ 20)^{\mathrm{T}}.$$

Two feasible starting points

$$\boldsymbol{u}_0=(20,\ 10,\ 10,\ 10)^{\mathrm{T}},$$
$$\boldsymbol{u}_0{}'=(15,\ 15,\ 10,\ 15)^{\mathrm{T}},$$

are used in the test. We use projective dynamic method to solve this problem. In our experiment, we take the starting points $\boldsymbol{u}_0$ in Fig.2 and in Fig.3. In Fig.2 and 3, we describe the trajectories of $x_1, x_2, y_1, y_2, \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}$ and $E(\boldsymbol{u})$, respectively.

From Figs. 2 and 3, we clearly see that x_1 converges to 20 and x_2 converges to 20. The merit function $E(\boldsymbol{u})$ converges to zero as t tends larger and larger. Numerical results show that projective dynamic method can generate the optimal solution to linear programming.

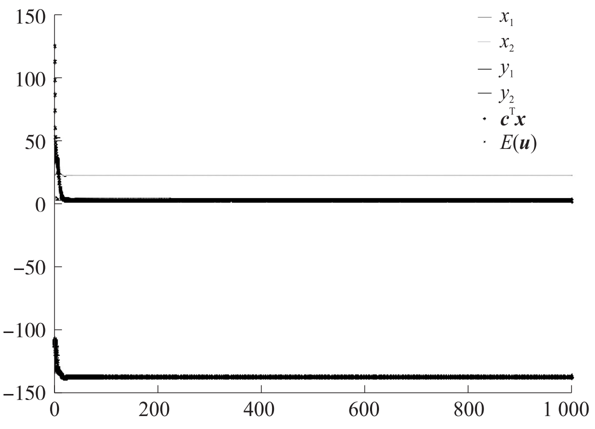

Fig.2　Transient behaviors of $x_1, x_2, y_1, y_2, \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}$ and $E(\boldsymbol{u})$ in Example 1 with starting point $\boldsymbol{u}_0$

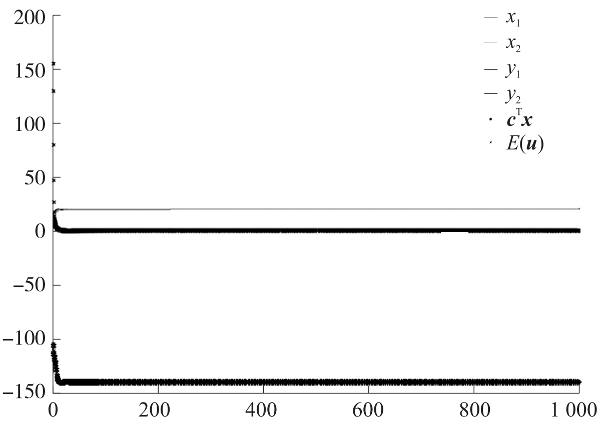

Fig.3　Transient behaviors of $x_1, x_2, y_1, y_2, \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}$ and $E(\boldsymbol{u})$ in Example 1 with starting point $\boldsymbol{u}'_0$

Consider a linear programming example used in [6, 7] as following

Example 2

$$\begin{aligned} \min \quad & -4x_1 - x_2 \\ s.t. \quad & x_1 - x_2 \leqslant 2, \\ & x_1 + 2x_2 \leqslant 8, \\ & x_i \geqslant 0,\ i = 1,2. \end{aligned}$$

Let

$$\boldsymbol{A} = \begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix},\ \boldsymbol{b} = \begin{pmatrix} 2 \\ 8 \end{pmatrix} \text{ and } \boldsymbol{c} = \begin{pmatrix} -4 \\ -1 \end{pmatrix}.$$

The optimal solution of this problem is $\boldsymbol{x}^* = (4, 2)^{\mathrm{T}}$, the minimal objective value is $\boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}^* = -18$. We test this problem by projective dynamic method.

The linear programming problem is equivalent to a linear variational inequality problem. We use the denotations as follows

$$\boldsymbol{u} = \begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{pmatrix}, \boldsymbol{M} = \begin{pmatrix} \boldsymbol{0} & \boldsymbol{A}^{\mathrm{T}} \\ -\boldsymbol{A} & \boldsymbol{0} \end{pmatrix}, \boldsymbol{q} = \begin{pmatrix} \boldsymbol{c} \\ \boldsymbol{b} \end{pmatrix},$$

and

$$\Omega = \left\{ \boldsymbol{u} = \begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{pmatrix} \middle| \boldsymbol{x} \in R_+^2,\ \boldsymbol{y} \in R_+^2 \right\}.$$

The optimal solution of this problem is $\boldsymbol{u}^* = \left(4, 2, \frac{7}{3}, \frac{5}{3}\right)^{\mathrm{T}}$.

Two feasible starting points $\boldsymbol{u}_0 = \text{zeros}(4, 1)$ and $\boldsymbol{u}_0' = 3*\text{ones}(4, 1)$ are used in the test.

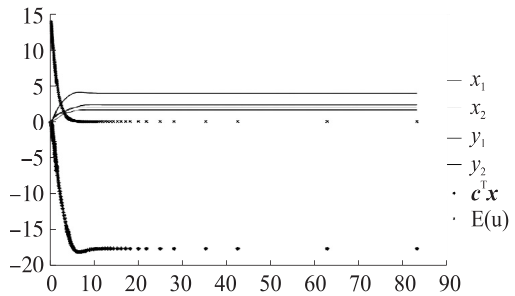

Fig.4 Transient behaviors of $x_1, x_2, y_1, y_2, c^{\mathrm{T}}x$ and $E(u)$ in Example 2 with starting point u'_0

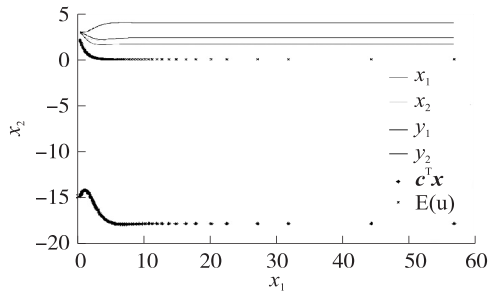

Fig.5 Transient behaviors of $x_1, x_2, y_1, y_2, c^{\mathrm{T}}x$ and $E(u)$ in Example 2 with starting point u'_0

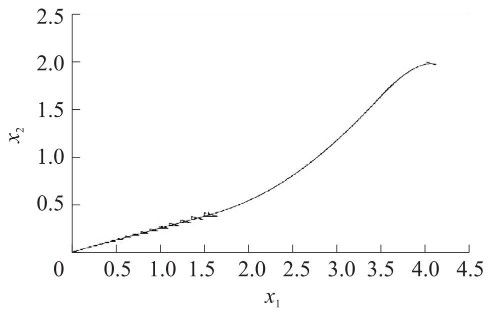

Fig.6 Transient behaviors of x_1, x_2 in Example 2 with starting point u_0

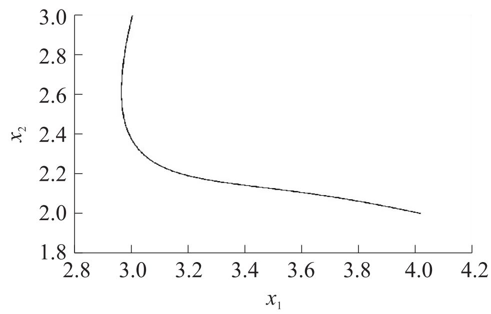

Fig.7 Transient behaviors of x_1, x_2 in Example 2 with starting point u'_0

In Fig. 4 and 5, we describe the trajectories of $x_1, x_2, y_1, y_2, \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}$ and $E(\boldsymbol{u})$, respectively. From Fig. 4, 5, 6 and 7, we clearly see that x_1 converges to 4 and x_2 converges to 2. The merit function $E(\boldsymbol{u})$ decreases monotonically as t tends larger and larger. Numerical results show that projective dynamic method can generate the optimal solution to linear programming.

（下转第35页）

[参考文献]

[1] DICKSON L E. History of the theory of numbers: diophantine analysis[M]. New York: Chelsea Publishing Co., 1966.
[2] HUA L K. Introduction to number theory[M]. Berlin: Springer-Verlag, 1982.
[3] NIVEN I, ZUCKERMAN H S, MONTGOMERY H L. An introduction to the theory of numbers[M]. 5th ed. New York: John Wiley & Sons, Inc, 1991.
[4] PONNUSAMY S, SILVERMAN H. Complex variables with applications[M]. Berlin: Birkhauser Boston, 2006.
[5] TRIPATHI A. The number of solutions to $ax+by=n$ [J]. Fibonacci quart, 2000(38): 290-293.
[6] WILF H S. Generating function ology[M]. 3rd ed. Wellesley: A K Peters, Ltd, 2006.

[责任编辑:黄　敏]

(上接第 13 页)

4 Conclusion

In this paper, a projective dynamics is proposed for minimizing general linear programming. The new method is based on the variational inequality properties. We extend the variational inequality method to construct a new ODE system. The new dynamic will be very useful to solve large scale optimization problems.

[参考文献]

[1] FLETCHER R. Practical methods of optimization[M]. 2nd ed. New York: John Wiley and Sons, 1987.
[2] LIAO L Z. A continuous method for convex programming problems[J]. J Optim Theory and Appl, 2005(124): 207-226.
[3] HE B S. A modified projection and contraction method for a class of linear complementarity problems[J]. J Comput Math, 1996(14): 54-63.
[4] EAVES B C. On the basic theorem of complementarity[J]. Math Programming, 1971(1): 68-75.
[5] HARKER P T, PANG J S. A damped-Newton method for the linear complementarity problem[M]//Computational Solution of Nonlinear Systems of Equations: Lectures in Applied Mathematics. USA: American Mathematical Society, 1990: 265-284.
[6] CICHOCKI A, UNBEHAUEN R. Neural networks for optimization and signal processing[M]. London, UK: Wiley, 1993.
[7] HAN Q M, LIAO L Z, QI H D, et al. Stability analysis of gradient-based neural networks for optimization problems[J]. J Global Optim, 2001(19): 363-381.

[责任编辑:黄　敏]