Almost-hamiltonicity, Neighborhood Intersections and Partially Square Graphs

Wu Zhengsheng, Zhou Xinghe

(School of Mathematics and Computer Science, Nanjing Normal University, Nanjing, 210097, PRC)

[摘要] Let G be a graph, and G^* the partially square graph of G. In this paper, by using the technique of the vertex insertion on $(k+2)$-connected $(k \geqslant 2)$ graphs, we give some sufficient conditions for graphs, which are to be 1-almost-hamiltonian or almost-hamiltonian-connected expressed by weighted sums of the neighborhood intersections in G of independent sets in G^*, where the weights are LTW-sequences.

[关键词] almost hamiltonnicity; partially square graphs; LTW-sequences

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0 Introduction

In this paper, the terminology and notations not defined will follow [5], and we consider simple finite graphs only. G will always stand for a graph.

A cycle C in G is called a dominating cycle (or D-cycle for simplicity) if $V(G) \setminus V(C)$ is an independent set of G; a path P in G is called a dominating path (or D-path for simplicity) if $V(G) \setminus V(P)$ is an independent set of G. A graph G is called almost-hamiltonian if every longest cycle of G is D-cycle; a graph G is called 1-almost-hamiltonian if $G - v$ is almost-hamiltonian for any $v \in V(G)$; a graph G is called almost-hamiltonian-connected if every longest (u, v)-path of G is D-path for any $\{u, v\} \subseteq V(G)$.

Let $t > 1$ be an integer. Denote

$$I_t(G) = \{Z : Z \text{ is an independet set of } G, |Z| = t\}.$$

Let $Z \subseteq V(G)$, and $|Z| = t$. For each $i \in \{0, 1, 2, \cdots, t\}$, denote

$$S_i(Z) = \{v \in V(G) : |N(v) \cap Z| = i\}, \text{and } s_i(Z) = |S_i(Z)|.$$

This is the concept of neighborhood intersections introduced in [8].

Let G be connected and $v \in V(G)$. Denote dist $(v, Z) = \min\limits_{z \in Z} \{\text{dist}(v, z)\}$ (where dist(v, z) stands for the distance between v and z),

$$N_i(Z) = \{v \in V(G) : \text{dist}(v, Z) = i\} \quad (i = 0, 1, 2, \cdots), \text{and}$$

$$n(Z) = |N_0(Z) \cup N_1(Z) \cup N_2(Z)| = |\{v \in V(G) : \text{dist}(v, Z) \leqslant 2\}|.$$

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Biography: Wu Zhengsheng, born in 1935, Professor at School of Mathematics and Computer Science of Nanjing Normal University. Research interest is graph theory.

Author's corresponding address: Zhou Xinghe, born in 1950, Deputy professor at School of Mathematics and Computer Science of Nanjing Normal University. Research interest is graph theory.

For $k \geqslant 1$, a non-negative rational sequence $(a_1, a_2, \cdots, a_{k+1})$ is called an LTW-sequence introduced in [6] (i.e., AS-sequence introduced in [3]) if

(1) $a_1 \leqslant 1$; and

(2) for arbitrary $i_1, i_2 \cdots, i_h \in \{2, 3, \cdots, k+1\}$, $\sum_{j=1}^{h} i_j \leqslant k+1$ implies $\sum_{j=1}^{h}(a_{i_j} - 1) \leqslant 1$.

The concepts of neithborhood intersections and LTW-sequence are of basic importance in study on hamiltonicity of graphs.

For $v \in V(G)$, denote $N[v] \cup \{v\}$. Let $\{u, v\} \subseteq V(G)$. Set

$J(u, v) = \{w \in N(u) \cap N(v) : N(w) \subseteq N[u] \cup N[v]\}$.

A. Ainouche and M. Kouider introduced the following concepts in [2]. The partially square graph G^* of G is a graph satisfying $V(G^*) = V(G)$ and $E(G^*) = E(G) \cup \{uv : uv \notin E(G)$, and $J(u, v) \neq \emptyset\}$.

[11] improves the results in [2], and gives the following theorems (In this paper, we always assume $(a_1, a_2, \cdots, a_{k+1})$ to be an LTW-sequence):

Theorem 1[11] Let G be a $(k+1)$-connected graph with $k \geqslant 2$, If $\sum_{i=1}^{k+1} a_i s_i(Z) > n(Z)$ in G for each $Z \in I_{k+1}(G^*)$, then G is 1-hamiltonian.

Theorem 2[11] Let G be a $(k+1)$-connected graph with $k \geqslant 3$. If $\sum_{i=1}^{k+1} a_i s_i(Z) > n(Z)$ in G for each $Z \in I_{k+1}(G^*)$, then G is hamiltonian-connected.

In this paper, we will prove the following new results by using the vertex inserting lemmas introduced in [7].

Theorem 3 Let G be a $(k+2)$-connected graph with $k \geqslant 2$. If $\sum_{i=1}^{k+1} a_i s_i(Z) + s_{k+1}(Z) > n(Z) + k + 1$ in G for each $Z \in I_{k+1}(G^*)$, then G is 1-almost-hamiltonian.

Theorem 4 Let G be a $(k+2)$-connected graph with $k \geqslant 3$. If $\sum_{i=1}^{k+1} a_i s_i(Z) + s_{k+1}(Z) > n(Z) + k + 1$ in G for each $Z \in I_{k+1}(G^*)$, then G is almost-hamiltonian-connected.

For Theorems 3 and 4, we give the following conjectures:

Conjecture 5 Let G be a $(k+1)$-connected graph with $k \geqslant 2$. If $\sum_{i=1}^{k+1} a_i s_i(Z) + s_{k+1}(Z) > n(Z) + k + 1$ in G for each $Z \in I_{k+1}(G^*)$, then G is 1-almost-hamiltonian.

Conjecture 6 Let G be a $(k+1)$-connected graph with $k \geqslant 3$. If $\sum_{i=1}^{k+1} a_i s_i(Z) + s_{k+1}(Z) > n(Z) + k + 1$ in G for each $Z \in I_{k+1}(G^*)$, then G is almost-hamiltonian-connected.

Sometimes, by a slight abuse of notation, we shall use the same letter for a subgraph (of G) and its vertex set, provided no ambiguity arises.

Let U and R be subgraphs of G (or subsets of $V(G)$), denote

$N(U) = \bigcup_{v \in U} N(v)$, and $N_R(U) = N(U) \cap R$.

Each cycle or path of G discussed in this paper will be assigned an orientation. A (u,v)-path is a path joining u and v having orientation from u to v. Let B be a cycle or a path of G, $\{x,y\}\subseteq V(B)$ (when B is a path, suppose that x appears no later than y), denote by $B[x,y]$ the oriented (x,y)-path of B (where the orientation was taken from B), $B(x,y]=B[x,y]-\{x\}$, $B[x,y)=B[x,y]-\{y\}$, and $B(x,y)=B[x,y]-\{x,y\}$. The reverse oriented graph of B is denoted by $\bar{B}$. Therefore, an oriented (y,x)-path of B will be denoted by $\bar{B}[y,x]$, and similarly for the others.

Let H be a connected subgraph of G. $\{x,y\}\subseteq N(H)\setminus V(H)$, denote by xHy one of the longest (x,y)-paths with all its internal vertices in H.

1 The vertex inserting lemmas and the other lemmas

In this section, we always assume that G is a connected non-hamiltonian graph and C is a maximal cycle of G (i.e., there is no cycle C' in G, such that $V(C)\subset V(C')$), and H is a component of $G-V(C)$. Assume also $\{v_1,v_2,\cdots,v_m\}\subseteq N_C(H)$ and $v_1,v_2,\cdots,v_m$ occur on C in the order of their indices. The subscriptions of v_i's will be taken modulo m. If $x\in V(C)$, denote by x^+ and x^- the successor and the predecessor of x along the orientation of C, respectively.

For $i\in\{1,2,\cdots,m\}$, if $u\in C(v_i,v_{i+1})$ and there is some $w\in C[v_{i+1},v_i)$ such that $\{w,w^+\}\subseteq N(u)$, then u is called an insertible vertex[7] in C with respect to $C(v_i,v_{i+1})$ (or simply say u is insertible).

In spite of there are some slight differences between the following Lemmas 1 – 9 and the correspondent Lemmas in [7], [9], [10] or [2], the proofs of them will be omitted since they are almost the same.

Lemma 1[7] Let $u\in C(v_i,v_{i+1})$ for some $i\in\{1,2,\cdots,m\}$. If all the vertices in $C(v_i,u)$ are insertible, then

(1) there exists a (u,v_i)-path P, such that $V(P)=V(C)$;

(2) $u\notin N_C(H)$, therefore there exists a vertex in $C(v_i,v_{i+1})$, which is not insertible.

By lemma 1(2), for each $i\in\{1,2,\cdots,m\}$, let x_i be the first non-insertible vertex in $C(v_i,v_{i+1})$.

Lemma 2[7] (1) If $u\in N_C(H)$, then $u^+\notin N_C(H)$;

(2) If $u\in N(x_i)\cap C[v_{i+1},v_i)$, then $u^+\notin N(x_i)$.

Lemma 3[7] For $\{i,j\}\subseteq\{1,2,\cdots,m\}$, if $y_i\in C(v_i,x_i]$, $y_j\in C(v_j,x_j]$, then

(1) there is no (y_i,y_j)-path Q with all its internal vertices not in $V(C)$;

(2) there is no $w\in C(x_i,v_j)$, such that $\{y_jw,y_iw^+\}\subseteq E(G)$.

Lemma 4[7] If $u\in N_C(H)\setminus\{v_1,v_2,\cdots,v_m\}$, $y\in\bigcup_{j=1}^{m}C(v_j,x_j]$, then $u^+y\notin E(G)$.

Let $X_M=\{x_0,x_1,\cdots,x_m\}$, where x_0 is an arbitrary vertex of H. Set $X\subseteq X_M$ such that $x_0\in X$, and $|X|=k+1\leqslant m+1$. $X\setminus\{x_0\}=\{x_{p_1},x_{p_2},\cdots,x_{p_k}\}$ (where $1\leqslant p_1<p_2<\cdots<p_k\leqslant m$). The subscriptions of p_j's will be taken modulo k.

Lemma 5[7] $X_M\in I_{m+1}(G)$, $X\in I_{k+1}(G)$.

Cycle C is said satisfying D-condition with respect to X, if for any $z^-\in S_{k+1}(X)\cap(V(C)\setminus$

$\{v_{p_1}, v_{p_2}, \cdots, v_{p_k}\}$) (we may assume $z^- \in C(v_{p_t}, v_{p_{t+1}})$), we always have $z \in S_0(X)$; and $z^+ \in S_0(X) \cup \{v^+_{t+1}\}$, or $z^+ \in S_1(X)$ and $z^{++} \in S_0(X) \cup \{v^+_{t+1}\}$.

A segment $C[z_1, z_2)$ ($\subseteq C[x_{p_t}, v_{p_{t+1}}]$, $t \in \{1, 2, \cdots, k\}$) is called a CX-segement if

(1) $C(z_1, z_2) \cap S_0(X) = \emptyset$, and

(2) $z_1 \in N_2(X) \cup X$, $z_2 \in S_0(X) \cup \{v^+_{p_{t+1}}\}$.

A CX-segment $C[z_1, z_2)$ is said to be simple if $C(z_1, z_2) \subseteq S_1(X)$; a simple CX-segment $C[z_1, z_2)$ is said to be D-simple if $z_1^- \in S_{k+1}(X)$; a simple CX- segment $C[z_1, z_2)$ is said to be ND-simple if $z_1^- \notin S_{k+1}(X)$.

Let $(a_1, a_2, \cdots, a_{k+1})$ be an LTW-sequence. Denote $\sigma_D(X) = \sum_{i=1}^{k+1} a_i s_i(X) + s_{k+1}(X)$.

Let $\xi = |\{v_{p_1}, v_{p_2}, \cdots, v_{p_k}\} \cap S_{k+1}(X)|$ and λ be the number of ND-simple CX- segments on C.

Lemma 6[7,9] If C satisfies D-condition with respect to X, then $\sigma_D(X) \leqslant n(X) + \xi - 1 - \lambda$, where $\xi \leqslant k$.

Now we cosider X_M and X in the partially square graph G^* of G.

Lemma 7[2] $X_M \in I_{m+1}(G^*)$, and therefore $X \in I_{k+1}(G^*)$.

Lemma 8 and § 2 will involve a graph G' other than G. In order to distinguish the notations such as $N(v)$, $N[v]$, $N(U)$, $J(u, v)$, $S_i(Z)$, $s_i(Z)$, $N_j(Z)$, $n(Z)$, $\sigma_D(Z)$ introduced for G, we will simply add a prime to the notations with respect to G'. For example, $N'(v)$, $N'[v]$, $\cdots$, $\sigma'_D(Z)$.

Lemma 8[10] Suppose that G' is a graph, $W \subseteq V(G')$, and $G = G' - W$. If $Z \in I_t((G')^*)$ and $Z \subseteq V(G)$, then $(V(G) \setminus N'_G(W)) \cap J(z_i, z_j) = \emptyset$ for each $\{z_i, z_j\} \subseteq Z$. So $Z \in I_t(G^*)$ if $N'_G(W) \cap J(z_i, z_j) = \emptyset$ for any $\{z_i, z_j\} \subseteq Z$.

Now we cosider that C satisfies D-condition with respect to X.

Lemma 9[7] Suppose that

(ⅰ) $|V(H)| > 1$, $X = \{x_0, x_{p_1}, x_{p_2}, \cdots, x_{p_k}\}$ ($x_0 \in V(H)$);

(ⅱ) there exists some $q \in \{1, 2, \cdots, k\}$ such that $p_{p-1} = p_q - 1$, $C(v_{p_{q-1}}, v_{p_q}) \cap N(x_0) = \emptyset$, there is a (x_0, v_q)-path Q (where $V(Q - v_q) \subseteq V(H)$), and $\{v_{p_1}, v_{p_2}, \cdots, v_{p_k}\} \setminus \{v_{p_q}\} \subseteq N(H - V(Q - v_q))$;

(ⅲ) for $z^- \in S_{k+1}(X) \setminus \{v_{p_1}, v_{p_2}, \cdots, v_{p_k}\}$, there is no cycle C' in G such that $V(C') \supseteq V(C) \setminus \{z\}$ and $|V(C')| > |V(C)|$.

Then C satisfies D-condition with respect to X.

2 Proofs of the theorems

Proof of theorem 3 By contradction. Suppose that there exists a $w \in V(G)$ such that there is a longest cycle C in $G' = G - w$, but C is not a D-cycle. Then there is some component H of $R = G' - V(C)$ such that $|V(H)| \geqslant 2$. Let $N_C(H) = \{v_1, v_2, \cdots, v_m\}$. For $i \in \{1, 2, \cdots, m\}$, choose x_i the first non-insertible vertex in $C(v_i, v_{i+1})$. Set $X_M = \{x_0, x_1, \cdots, x_m\}$, where x_0 is an arbitrary vertex of H. We first prove three results.

(a) If $X_M \notin I_{m+1}(G^*)$, then there is some $x_l \in X_M \setminus \{x_0\}$ such that $X_M \setminus \{x_l\} \in I_m(G^*)$.

In fact, by Lemma 7, $X_M \in I_{m+1}((G')^*)$. Thus, there are $\{x_l, x_{l'}\} \subseteq X_M$ such that $J(x_l, x_{l'}) \neq \{w\}$. Without loss of generality, we may assume that $x_l \neq x_0$.

Suppose that $X_M \setminus \{x_l\} \notin I_m(G^*)$, then there is some $\{x_i, x_j\} \subseteq X_M \setminus \{x_l\}$ such that $J(x_i, x_j) \neq \emptyset$. Also $J(x_l, x_{l'}) \neq \emptyset$. Thus $J(x_i, x_j) = J(x_l, x_{l'}) = \{w\}$. So, it is easy to see that $x_l \in N(x_i) \cup N(x_j)$, contradicting Lemma 5.

(b) For a fixed $v_l \in N_C(H)$, there exist some $x_0 \in V(H)$ and $\{v_q, v_{q+1}\} \subseteq N_C(H) \setminus \{v_l\}$ such that $v_q \in N(x_0)$ and $v_{q-1} \in N(H - x_0)$.

In fact, since G' is 3-connected, $|V(H)| \geqslant 2$ and x_0 is an arbitrary vertex of H, it is easy to see the result holds.

(c) there exists some $X \subseteq X_M$ such that $X \in I_{k+1}(G^*)$ and C satisfies D-condition with respect to X.

In fact, by (a), $X_M \setminus \{x_l\} \in I_m(G^*)$ and $l \neq 0$. By (b), We may choose $x_0, x_{q-1}, x_q \in X$. Also may choose $\{v_{p_1}, v_{p_2}, \cdots, v_{p_k}\} \setminus \{v_{p_{q-1}}, v_{p_q}\} \subseteq N(H - x_0) \setminus \{v_{p_{q-1}}, v_l\}$ since $m \geqslant k+1$. Thus $X = \{x_0, x_{p_1}, x_{p_2} \cdots, x_{pk}\} \in I_{k+1}(G^*)$, and the conditions (i) and (ii) of Lemma 9 are satisfied. Since C is a longest cycle G', the condition (iii) of Lemma 9 is satisfied. Thus by Lemma 9, this result holds.

Since $G' = G - w$, there is some $q \in \{0, 1, \cdots, k+1\}$ such that $w \in S_q(X)$. Then $s_q(X) = s'_q(X) + 1$, and $s_i(X) = s'_i(X)$ for each $i \in \{0, 1, \cdots, k+1\} \setminus \{q\}$. Let $n(X) = n'(X) + \mu$, where $\mu \in \{0, 1, \cdots\}$. Clearly, $\mu \geqslant 1$ if $q \neq 0$. By the definition of LTW-sequence, $a_i \leqslant 2$ for each $i \in \{1, 2, \cdots, k+1\}$. We obtain the following contradiction by (c) and Lemma 6

$$\sum_{i=1}^{k+1} a_i s_i(X) + s_{k+1}(X) = \sigma_D(X) \leqslant \sigma'_D(X) + 2 + \mu \leqslant n'(X) + k + 1 + \mu = n(X) + k + 1.$$

Proof of theorem 4 By contradiction. Suppose there exists some $\{u_1, u_2\} \subseteq V(G)$ such that there is a longest (u_1, u_2)-path in G which is not a D-path. Then there is sme component H of $R = G - V(P)$ such that $|V(H)| \geqslant 2$. Let $N_P(H) = \{v_1, v_2, \cdots, v_m\}$, where $m \geqslant k+2$. Add two new vertices w_1, w_2 and three new edges u_1w_1, w_1w_2, u_2w_2 to G and denote by G' the resulting graph. Then $C = P[u_1, u_2]w_1w_2u_1$ is a cycle in G'. Let the orientation of C agree with that of P. Clearly C is a maximal cycle in G'. By Lemma 1(2), set x_i as the first non-insertible vertex in $C(v_i, v_{i+1})$, $i \in \{1, 2, \cdots, m\}$. Let x_0 be an arbitrary vertex of $V(H)$ and $X_M = \{x_0, x_1, \cdots, x_m\}$. We first prove three results.

(a) If $X_M \setminus \{x_m\} \notin I_{m-1}(G^*)$. then $X_M \setminus \{x_1, x_m\} \in I_{m-1}(G^*)$.

In fact, by Lemma 8, there are x_l and $x_{l'}$ such that $J(x_l, x_{l'}) \subseteq \{u_1, u_2\}$. If $u_2 \in J(x_l, x_{l'})$, then by Lemma 2, $u_2^- \in N(u_2) \setminus N(x_l) \cap N(x_{l'})$, a contradiction; if $u_1 \in J(x_l, x_{l'})$ and $x_1 \notin \{x_l, x_{l'}\}$, then by Lemma 2, $u_1^+ \in N(u_1) \setminus N(x_l) \cap N(x_{l'})$, a contradiction. Thus $J(x_l, x_1) = \{u_1\}$.

Suppose that $X_M \setminus \{x_1, x_m\} \notin I_m(G^*)$. Then there is some $\{x_i, x_j\} \subseteq X_M \setminus \{x_1, x_m\}$ such that $J(x_i, x_j) = \emptyset$. Also $J(x_l, x_1) \neq \emptyset$. Thus $J(x_i, x_j) = J(x_l, x_1) = \{u_1\}$. So it is easy to see that $x_1 \in N(x_i) \cup N(x_j)$, contradicting Lemma 5.

(b) There exist some $x_0 \in V(H)$ and $\{v_q, v_{q+1}\} \subseteq N_C(H) \setminus \{v_1, v_m\}$ such that $v_q \in N(x_0)$ and $v_{q-1} \in N(H - x_0)$.

In fact, since G is 4-connected, $|V(H)|\geq 2$ and x_0 is an arbitrary vertex of H, it is easy to see that this result holds.

(c) there exists some $X\subseteq X_M$ such that $X\in I_{k+1}(G^*)$ and C satisfies D-condition with respect to X.

In fact, by (a), we may assume that $X_M\setminus\{x_1,x_m\}\in I_{m-1}(G^*)$. By (b), We may choose x_0, x_{q-1}, $x_q\in X$, Without loss of generality, we may assume that $x_{q-1}=x_{p_{q-1}}$ and $x_q=x_{p_q}$. Also we may choose $\{v_{p_1},v_{p_2},\cdots,v_{p_k}\}\setminus\{v_{p_{q-1}},v_{p_q}\}\subseteq N(H-x_0)\setminus\{v_{p_{q-1}},v_1,v_m\}$ since $m\geq k+2$. Thus $X=\{x_0,x_{p_1},x_{p_2},\cdots,x_{p_k}\}\in I_{k+1}(G^*)$, and the conditions (ⅰ) and (ⅱ) of Lemma 9 are satisfied. By the choice of y_m, $u_2^-\notin S_{k+1}(X)\setminus\{v_{p_1},v_{p_2},\cdots,v_{p_k}\}$. Since P is a longest (u_1,u_2)-path in G and $u_1^-=w_1$, $u_2^-\notin S_{k+1}(X)\setminus\{v_{p_1},v_{p_2},\cdots,v_{p_k}\}$, the condition (ⅲ) of Lemma 9 is satisfied by the construction of G'. Thus by Lemma 9, this result holds.

By the construction of G', $n'(X)\leq n(X)+2$, and $w_1,w_2\in S'_0(X)$. Thus $s_i(X)=s'_i(X)$ for each $i\in\{1,2,\cdots,k+1\}$.

$$\sum_{i=1}^{k+1}a_is_i(X)+s_{k+1}(X)=\sum_{i=1}^{k+1}a_is'_i(X)+s'_{k+1}(X)=\sigma'_D(X)\leq n'(X)+k-1$$
$$\leq n(X)+k+1.$$

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几乎 Hamilton 连通图和部分平方图

吴正声，周兴和

（南京师范大学数学与计算机科学学院，南京，210097）

[摘要] G 为图，G^* 是 G 的部分平方图．运用（$k+2$）连通图（$k\geqslant 2$）上的插点技术，借助 LTW 序列对 G^* 中独立集的邻域交加权，证明了图 G 是几乎 Hamilton 连通的一些充分条件．

[关键词] 几乎 Hamilton 性；部分平方图；LTW 序列

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