Curie Temperature and Magnetization Profiles of Quasiperiodic Layer Quantum Ising Model

Liu Fangai, Lei Shuguo

(College of Science, Nanjing University of Technology, 210009, Nanjing, China)

Abstract: Under the approximation of the mean field, Curie temperature and magnetization profiles of layer quantum Ising model with a transverse field were studied. The Curie temperature equation of the model was derived. The dependence of Curie temperature on the Fibonacci number n and the external field is given. It is found that the Curie temperature of quasiperiodic models is like to that of periodic models if the chain is long enough. With the same framework, the magnetization profiles of the system was also calculated. The method proposed here can be applied to other quasiperiodic model.

Key words: Curie temperature, quasiperiodic, quantum Ising model, magnetization

CLC number O441, **Document code** A, **Article ID** 1001-4616(2004)02-0046-05

0 Introduction

Phase transition and critical phenomena are very active questiones in the field of condensed matter physics, among which paramagnetic and ferromagnetic phase transition are mostly concentrated. Much attention has been paid to them and a large quantity of achievements have been got concerning to the paramagnetic and ferromagnetic phase transition and Curie temperature of the magnetic material[1]. Also there is much interest in the magnetic properties of the surface effect[2]. Recently, the effects of multi-surface modification on Curie temperature have been studied and the Curie temperature equation is derived[3]. On the other hand, considerable interest has been focused on the physics property of the quasiperiodic(QP) system since the quasicrystal with 5-symmetry axis was found in the MnAl alloy[4] and the superlattice with quasicrystal was preparationed in the lab[5]. A lot of experimental and theoretical worker show great interest in it. Kohmoto and co-workers[6,7] extensively studied the spectral property of one-dimensional quasiperiodic crystals. Further more, the spectral and magnetic properties of the quantum Ising model with nearest-neighbor couplings ordered according to different aperodic sequence have been investigated[8,9].

Quasiperiodic system lies in between periodic and random system, and shows much different properties. The study on phase transition and Curie temperature are consequently a wonderful thing.

1 Models and method

The model studied in the paper is the Ising model with a transverse field. The Hamiltonian[3] is general described by

$$H = -\frac{1}{2}\sum_{(i,j)}\sum_{(r,r')} J_{ij} s_{ir}^{z} s_{jr'}^{z} - \sum_{i,r} \Omega_i s_{ir}^{x} \tag{1}$$

where s_{ir}^{x} and s_{ir}^{z} are x and z components of the pseudo-spin, (i,j) are plane indices and (r,r') are different sites of the planes. J_{ij} indicate the exchange constants. We assume the transverse field Ω_i is dependent only on the layer index and consider the interaction between neighboring sites. In addition, we take $\Omega_i = \Omega$ for simplicity.

Under the mean field approximation, the spin average $\langle \boldsymbol{s}_i \rangle$ can be described below

$$\langle \boldsymbol{s}_i \rangle = \frac{\boldsymbol{H}_i}{2|\boldsymbol{H}_i|}\tanh\left(\frac{|\boldsymbol{H}_i|}{2k_B T}\right) \tag{2}$$

Received date: 2003 - 09 - 02.

Biography: Liu Fangai, female, born in 1975, Assistant of Nanjing University of Technology, Majored in numerical calculation of low dimension spin system, E - mail: faliu@sohu.com

where $H_i(\Omega, 0, \sum_j J_{ij}\langle s_j^z\rangle)$ is the mean field acting on the ith spin, k_B is the Boltzmann constant and T is temperature. At a temperature close to or bellow the Curie temperature, $\langle s_i^x\rangle$ and $\langle s_j^z\rangle$ are small, $H_i \approx \Omega_i = \Omega$, Eq(2) can be approximationed as

$$\langle s_i^x\rangle = (1/2)\tanh(\Omega/(2k_BT)) \tag{3}$$

$$\langle s_i^z\rangle = (1/2)\tanh(\Omega/(2k_BT))[z_0 J_{ii}\langle s_i^z\rangle + z(J_{i,i+1}\langle s_{i+1}^z\rangle + J_{i,i-1}\langle s_{i-1}^z\rangle)] \tag{4}$$

z_0 and z are numbers of nearest neighbors in a certain plane and between successive planes respectively.

Using transfer matrix method, Eq(4) can be rewritten bellow

$$\begin{pmatrix} m_{i+1} \\ m_i \end{pmatrix} = \boldsymbol{M}_i \begin{pmatrix} m_i \\ m_{i-1} \end{pmatrix} \tag{5}$$

where the transfer matrix $\boldsymbol{M}_i$ is defined by

$$\boldsymbol{M}_i = \begin{pmatrix} \tau - \dfrac{z_0 J_{i,i}}{zJ_{i,i-1}} & -\dfrac{J_{i,i-1}}{J_{i,i+1}} \\ 1 & 0 \end{pmatrix} \tag{6}$$

and

$$m_i = \langle s_i^z\rangle, \tau = \frac{2\Omega}{zJ}\coth\frac{\Omega}{2k_BT} = \frac{2}{z}\omega\coth\frac{\omega}{T'} \tag{7}$$

We call the reduced Curie temperature $T'(=\frac{2k_BT}{J})$ Curie temperature for short in the context. In the same way, we shorten $\omega(=\frac{\Omega}{J})$ for the external field.

We consider the Fibonacci layer quantum system constituted by two kinds of materials marked with A and B. The general Fibonacci system is defined by the follow substitution rule:

$$A \to A' = A^m B^n \qquad B \to B' = B^p A^q \tag{8}$$

For the sick of simplicity, we assume the interaction between successive layer of the system are same, i.e., $J_{AB} = J_{BA} = J_{AA} = J_{BB}$, the interactions in one layer are J_A and J_B respectively. Then the transfer matrix Eq(6) has only two forms:

$$\boldsymbol{M}_A = \begin{pmatrix} \tau - j_A & -1 \\ 1 & 0 \end{pmatrix}, \quad \boldsymbol{M}_B = \begin{pmatrix} \tau - j_B & -1 \\ 1 & 0 \end{pmatrix} \tag{9}$$

here $j_A = \frac{z_0 J_A}{zJ}$, and $j_B = \frac{z_0 J_B}{zJ}$.

We start with the general Fibonacci sequence model, $m=1, n=1, p=1, q=0$, that is

$$A \to AB \qquad B \to A \tag{10}$$

The layer system generated by the rule is

$$ABAAABABA\cdots$$

In the model, $\sigma = \lim_{l\to\infty}\frac{F_l}{F_{l-1}} = \frac{1+\sqrt{5}}{2}$ is called golden mean, and F_l, called Fibonacci number, is the total number of letters in the l th iteration of the word A of the transformation.

According to the construction rule Eq(10), we can derive the lth transfer matrix for the standard Fibonacci sequence system without difficulty

$$\boldsymbol{M}_l = \boldsymbol{M}_{F_l} = \boldsymbol{M}_{l-2}\boldsymbol{M}_{l-1}, \boldsymbol{M}_1 = \boldsymbol{M}_A, \boldsymbol{M}_2 = \boldsymbol{M}_B \tag{11}$$

Now we consider the F_n approximation of the quantum layer system. The complete quasiperiodic system is got with $n \to \infty$. Considering the periodic bounder condition, we easily get

$$\begin{pmatrix} m_1 \\ m_0 \end{pmatrix} = \boldsymbol{M}_n \begin{pmatrix} m_1 \\ m_0 \end{pmatrix} \tag{12}$$

or

$$\begin{cases}(M_{n1,1}-1)m_1+M_{n1,2}m_0=0\\ M_{n2,1}m_1+(M_{n2,2}-1)m_0=0\end{cases} \tag{13}$$

Up to now, we have got the universe Curie temperature of Fibonacci system. Eq(13) contains all the information about the Curie temperature. From above equation, the phase transition temperature can be calculated, then the magnetization profiles can be determined from Eq(5) if we know the magnetization m_0.

2 Results and discussion

In order to compare with the quasiperiodic layer quantum Ising system, we apply the method to the periodic condition firstly. Under the circumstance, we take $j_A=j_B$, that is, $\boldsymbol{M}_1=\boldsymbol{M}_2$, then the transfer matrix got form Eq(11) is

$$\boldsymbol{M}_n=\boldsymbol{M}_1^n=\boldsymbol{M}_2^n=\begin{pmatrix}\tau-j & -1\\ 1 & 0\end{pmatrix}^n \tag{14}$$

then

$$\mathrm{Tr}\boldsymbol{M}_n=\lambda_1^n+\lambda_2^n \tag{15}$$

here λ_1 and λ_2 are the eigenvalues of the matrix $\boldsymbol{M}_1(\boldsymbol{M}_2)$. Considering that Eq(15) is established for arbitrary n, then

$$\lambda_1=\lambda_2=1 \tag{16}$$

and $\lambda_{1,2}=\dfrac{(\tau-j)\pm\sqrt{(\tau-j)^2-4}}{2}$.

So we have

$$\tau-j=2 \tag{17}$$

and the Curie temperature

$$T'=\frac{\omega}{\ln\dfrac{(j+2)z+2\omega}{(j+2)z-2\omega}} \tag{18}$$

the external field is restricted

$$\omega<\frac{(j+2)z}{2} \tag{19}$$

Secondly, we discuss another periodic layer circumstance: $j_A\neq j_B$ while $J_1=J_3=J_5=\cdots=J_A$, $J_2=J_4=J_6=\cdots=J_B$, the transfer matrix

$$\boldsymbol{M}_n=\boldsymbol{M}_1\boldsymbol{M}_2\boldsymbol{M}_1\boldsymbol{M}_2\cdots\boldsymbol{M}_1\boldsymbol{M}_2=(\boldsymbol{M}_1\boldsymbol{M}_2)^{\frac{n}{2}}=\begin{pmatrix}(\tau-j_A)(\tau-j_B)-1 & -(\tau-j_A)\\ \tau-j_B & -1\end{pmatrix} \tag{20}$$

By means of the same diagonalize procedure, the Curie temperature equation then

$$\mathrm{Tr}\boldsymbol{M}_n=\lambda_1^{\frac{n}{2}}+\lambda_2^{\frac{n}{2}}=2 \tag{21}$$

here λ_1 and λ_2 are the eigenvalues of product matrix $(\boldsymbol{M}_1\boldsymbol{M}_2)$. Consequently the Curie temperature equation now becomes

$$(\tau-j_A)(\tau-j_B)=4 \tag{22}$$

In the fact these two periodic circumstances belong to one form:

$$\tau=c(\text{constant}) \tag{23}$$

corresponding Curie temperature

$$T'=\frac{\omega}{\ln\dfrac{cz+2\omega}{cz-2\omega}} \tag{24}$$

and the condition of phase transition

$$\omega < \frac{cz}{2} \tag{25}$$

From the expression of the Curie temperature of the model, it is found that the Curie temperature decreases successively with the external field. As is also seen in Fig.1, the solid line shows the first circumstance, and the dot, the second.

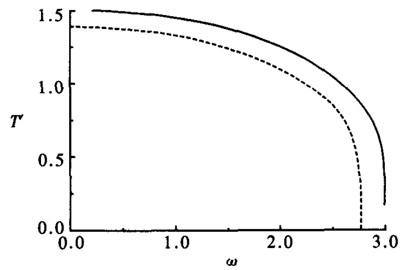
In Sec (1), we take the general Fibonacci sequence layer quantum Ising model as a sample to get the equation. Different from the periodic circumstance, the quasiperiodic layer quantum Ising system equation has not an analytical solution. As is found in Sec. (1) when we derive the Curie temperature equation. So in the paper, we apply numerical method to solve the equation.

During the course of numerical calculation, we find that there is plenty of roots under the condition that z, j_A, j_B and ω is given. Fig.2 shows the curve of the number of the root N varies with the Fibonacci number n. Although the number of root increases rapidly with n, the biggest root tends to a more and more fixed value. This must be a reflect of the physics property of the quasiperiodic system. With no doubt, the biggest root is the Curie temperature we wanted. Fig.3 shows the Curie temperature of the Fibonacci sequence quantum Ising model varies with the external field. Comparing with Fig.1 as is can be seen that the Curie temperature value at the same external field point is higher than the same condition as the periodic system while lower than the complete periodic system where $j_A = j_B = j = 1.0$. This can be inferred from that the composition of $A(j_A)$ is larger than that of $B(j_B)$.

After we got a certain Curie temperature, as we say in Sec (1), we use m_0 to scale the magnetization m_i, i.e., take m_0 as the unit of the magnetization. m_i is obtained from the boundary condition, as is seen in Eq(12) or Eq(13). Fig.4 shows the reduced magnetization m_i/m_0 dependence on the lattice site number i. In this figure, as i increases from 0, the magnetization sometimes increases, sometimes decreases. There has no explicit tendency for the reduced magnetization. As for it comes to 1 in the end just as the beginning, this satisfied the boundary condition very well.

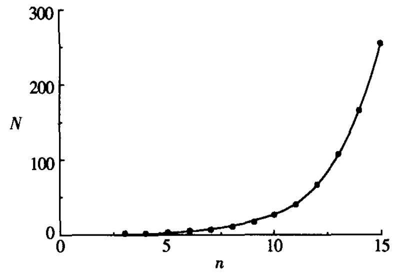
3 Conclusion

In brief, in this paper we study the property of Curie temperature and magnetization of quasiperiodic quantum layer Ising model, analytically or numerically got the solution of the Curie temperature equation. Moreover, using m_0 as the unit scale, we also obtain the magnetization profile of the system. The method can be used to any other quasiperiodic sequence system.



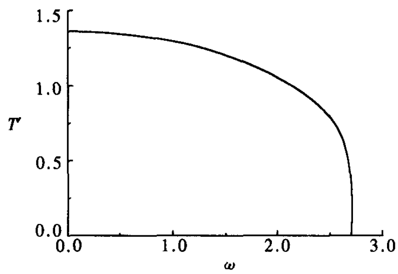
The parameters $z=2$, of the solid line, $j=1.0$, and the dot line, $j_A=1.0$, $j_B=0.5$

Fig.1 The dependence of the Curie temperature on the external field for the periodic model



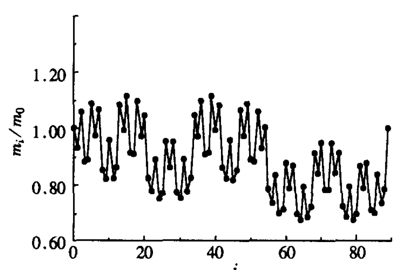
The parameters $\omega=1$, $z=2$, $j_A=1.0$, and $j_B=0.5$

Fig.2 The number of root changes with the Fibonacci number



The parameters $z=2$, $j_A=1$, $j_B=0.5$, and $n=11$

Fig.3 Same as Fig.1 for the Fibonacci sequence



The parameters $\omega=1$, $z=2$, $j_A=1.0$, $j_B=0.5$, and $n=11$

Fig.4 The reduced magnetization m_i/m_0 vs lattice site number i for Fibonacci sequence

[References]

[1] Callaway J. Quantun Theory of the Solid State[M]. New York: Academic Press, 1976.

[2] Saber M, Ainane A, Dujardin F, *et al*. Magnetic properties of a transverse spin-(1/2) Ising film[J]. Phys Rev B, 1999, 59(10): 6908.

[3] Wang Xiaoguang, Pan Shaohua, Yang Guozhen. Effects of multi-surface modification on Curie temperature of ferroelectric films[J]. J Phys Condens Matter, 1999, 11(34): 6581—6588.

[4] Shechtman D, Blech I, Gratias D, *et al*. Metallic Phase with Long-Range Orientational Order and No Translational Symmetry[J]. Phys Rev Lett, 1984, 53(20): 1951—1953.

[5] Merlin R, Bajema K, Royclarke, *et al*. Quasiperiodic GaAs-AlAs Heterostructures[J]. Phys Rev Lett, 1985, 55(17): 1768—1770.

[6] Benza V G. Phase transition in the generalized Fibonacci quantum Ising models[J]. Phys Rev B, 1990, 41(13): 9578.

[7] Kohmoto M, Kadanoff L P, Tang C. Localization Problem in One Dimension: Mapping and Escape[J]. Phys Rev Lett, 1983, 50(23): 1870—1872.

[8] Niu Q, Nori F. Renormalization-Group Study of One-Dimensional Quasiperiodic Systems[J]. Phys Rev Lett, 1986, 57(16): 2057—2060.

[9] Niu Q, Nori F. Theory of superconducting wire networks and Josephson-junction arrays in magnetic fields[J]. Phys Rev B, 1989, 39(4): 2134—2150.

准周期量子伊辛模型的居里温度和磁化强度

刘方爱，类淑国

(南京工业大学理学院，210009，江苏，南京)

[摘要] 利用平均场理论研究了一维横场中准周期层状量子伊辛模型性质，得到系统的居里温度方程，发现在自旋链足够长的情况下其居里温度回归到周期系统．并计算了系统的平均磁化强度随长度的变化．

[关键词] 居里温度，准周期，量子伊辛模型，磁化强度

[责任编辑：丁蓉]