The Solution of Nonlinear Complementarity Problems with Interval Gauss-Seidel Method

Pu Zhiqin[1], Xue Qiaoling[2], Shen Zuhe[3]

(1. School of Mathematics and Computer Science, Nanjing Normal University, 210097, Nanjing, China)
(2. Department of Mathematics, Nanjing University of Information Science and Technology, 210044, Nanjing, China)
(3. Department of Mathematics, Nanjing University, 210093, Nanjing, China)

Abstract: In this paper, we use Krawczyk-like interval operator and Gauss-Seidel interval iterative method to solve the nonlinear complementarity problems. This is another application of Krawczyk interval operator.

Key words: Krawczyk interval operator, Gauss-Seidel iterative, nonlinear complementarity problems
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用区间 Gauss-Seidel 方法解非线性互补问题

浦志勤[1]，薛巧玲[2]，沈祖和[3]

(1. 南京师范大学数学与计算机科学学院, 210097, 江苏, 南京)
(2. 南京信息工程大学数学系, 210044, 江苏, 南京)
(3. 南京大学数学系, 210093, 江苏, 南京)

[摘要] 在本文中,我们使用了 Krawczyk-like 区间算子和 Gauss-Seidel 区间算子方法解非线性互补问题. 这是 Krawczyk 区间算子的又一次应用.

[关键词] Krawczyk 区间算子,Gauss-Seidel 迭代,非线性互补问题

0 Introduction

Complementarity theory, which has been studied intensively in the last several decades, is generally considered to be a domain of applied mathematics. The complementarity problem arises in a variety of contexts such as optimization , game theory, economics, classical mechanics, stochastic optimal control, etc. The primary source of complementarity problems are equilibrium problems in economics, physics and engineering and the necessary conditions for optimality for mathematical programs.

Because of the many important applications of the complementarity problem, the development of the conditions assuring the existence of a solution to this problem was always of big interest. So far many researchers have established a variety of conditions for the solvability of the complementarity problem.

Our study is motivated by the papers by Zhenyu Wang and Zuhe Shen [5], and B. C. Eaves [4]. Zhenyu Wang and Zuhe Shen [5] introduced a Krawczyk-like interval operator.

Let f: $\mathbf{R}^n \to \mathbf{R}^n$. The nonlinear complementarity problem, denoted by $NCP(f)$, is to find the vector $x \in \mathbf{R}^n$ such that

$$x > 0, f(x) > 0, x^T f(x) = 0, \tag{1}$$

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Biography: Pu Zhiqin, born in 1965, Lecturer, Majored in the research of computational mathematics. E-mail: puzhiqin@njnu.edu.cn

where we assume that f is continuously differentiable.

For the problem many iterative method have been developed, but few inclusion method has been studied for it to give both the approximate solutions and the guaranteed error bounds synchronously before G. E. Alefeld, X. Chen and F. A. Potra proposed the validation methods for linear and nonlinear complementarity problems in [2,3]. However, just as we notice, the validation method are based on the Moore test for the existence of solutions to the following equivalent formulation

$$\min\{x, f(x)\} = 0, \tag{2}$$

and the author had to utilize the generalized Jacobian or the slope to construct the algorithms since the min map is non-smooth. The problem (1) can be formulated as a fixed point problem [4]. x solves $NCP(f)$ if and only if x is a fixed point of the map

$$P(x) = \max\{0, x - f(x)\}.$$

In this paper, we use Krawczyk-like interval operator and Gauss-Seidel interval iterative method to solve the nonlinear complementarity problems.

The paper is organized as follows. In the next section factorable functions is introduced. A method of the solution of the $NCP(f)$ is proposed in Section 3. An algorithm and some examples on several $NCP(f)$ are provided in Section 4 and Section 5.

The following notations is used throughout our paper. For an one-dimensional interval $[x] = [x_I, x_S]$, define

$$|[x]| = \max\{|x_I|, |x_S|\},$$
$$w([x]) = x_S - x_I,$$
$$m([x]) = (x_I + x_S)/2.$$

For an n-dimensional interval $[x] = ([x_1], [x_2], \cdots [x_n])^T$ define

$$|[x]| = \max_i\{|[x_i]|\},$$
$$w([x]) = \max_i\{w([x_i])\},$$
$$m([x]) = (m([x_1]), m([x_2]), \cdots, m([x_n]))^T.$$

For an interval matrix $[A]$ with interval coefficients $[A_{ij}]$, define

$$\|[A]\| = \max_i \sum_{j=1}^{n} |[A_{ij}]|,$$

and define $m([A])$ the real matrix with components $m([A_{ij}])$.

1 Factorable Functions

Derivative arithmetic and slope arithmetic are applicable to so-called factorable functions.

Definition 1.1 The function f: $D \subseteq \mathbf{R}^n \to \mathbf{R}^1$ is a factorable function if and only if it can be represented by an expression $f(x)$ which is the last element in a finite sequence $(f_i(x))$ of expressions such that

$$f_i(x) = x_i, \ (i = 1, \cdots, n)$$

and for $i > j$, k

$$f_i(x) = c_i \in \mathbf{R},$$

or

$$f_i(x) = -f_j(x),$$

or for $* \in \{+, -, \cdot, /\}$,

$$f_i(x) = f_j(x) f_k(x),$$

or for $\phi_i \in \Phi$,

$$f_i(x) = \varphi_i(f_j(x)),$$

where

$$\Phi = \{(\cdot)^{1/2}, \exp(\cdot), \ln(\cdot), \sin(\cdot), \cdots\cdots\}.$$

Theorem 1.2 [1] Let f: $D \subseteq \mathbf{R}^n \to \mathbf{R}^1$ be a factorable function defined by the sequence $f_i(x)$. Let the first order slope $f[x, z]$ of f be defined recursively by

$$f_i(x) = c_i \Rightarrow f_i[x, z] = 0, \tag{2}$$

$$f_i(x) = x_i \Rightarrow f_i[x, z] = e^{(i)} \qquad (i = 1, \cdots, n), \tag{3}$$

where $e^{(i)}$ is row i-th of the $n \times n$ unit matrix,

$$f_i(x) = -f_j(x) \Rightarrow f_i[x, z] = f_j[x, z], \tag{4}$$

$$f_i(x) = f_j(x) \pm f_k(x) \Rightarrow f_i[x, z] = f_j[x, z] \pm f_k[x, z], \tag{5}$$

$$f_i(x) = f_j(x)\, f_k(x) \Rightarrow f_i[x, z] = f_j[x, z] f_k(x) + f_j(z) f_k[x, z], \tag{6}$$

$$f_i(x) = f_j(x)/f_k(x) \Rightarrow f_i[x, z] = (f_j[x, z] - f_i(z) f_k[x, z])/f_k(x), \tag{7}$$

and for $\phi_i \in \Phi$,

$$f_i(x) = \phi_i(f_j(x)) \Rightarrow f_i[x, z] = f_i[f_j(x), f_j(z)] f_j[x, z], \tag{8}$$

where j, $k < i$. Then $(\forall x, z \in D)$

$$f(x) = f(z) + f[x, z](x - z). \tag{9}$$

The theorem (1.2) contains formulate for determining first order slopes of factorable functions. The first order slope $f[x, z]$ of f is the first order divided difference of f and may therefore be regarded as an estimate of $f'(x)$.

2 A Test for Existence of Solutions

2.1 Interval Max-Operation

Let $[x] = [x_I, x_S]$ be an n-dimensional interval vector, where x_I, $x_S \in \mathbf{R}^n$ and $x_I < x_S$. Define a component-wise interval operator

$$\max\{0, [x]\} = [\max\{0, x_I\}, \max\{0, x_S\}].$$

For example

$$[x] = \begin{pmatrix} [-2, 4] \\ [-2, -1] \end{pmatrix},$$

then

$$\max\{0, [x]\} = \begin{pmatrix} [0, 4] \\ 0 \end{pmatrix}.$$

It is easy to see that the operation might transform some omponents of the n-imensional interval vector $[x]$ as a point interval $[0, 0]$, which we denote by 0 for convenience in the rest of the paper.

Proposition 2.1 $\max\{0, x\} \in \max\{0, [x]\}$, $\quad \forall x \in [x]$.

2.2 Properties of Complementarity

As we know, the solution to the complementarity problem $NCP(f)$ is not affine invariant. On this issue, we just have the following result, which could be verified easily.

Proposition 2.2 Let $\boldsymbol{D} = \mathrm{diag}(d_1, d_2, \cdots, d_n)$ be a diagonal matrix, where $d_i > 0$, $i = 1, 2, \cdots, n$. Then x solves the problem $NCP(f)$ if and only if x solves $NCP(\boldsymbol{D} f)$.

Proposition 2.3 Let $\boldsymbol{D} = \mathrm{diag}(d_1, d_2, \cdots, d_n)$ be a diagonal matrix, where $d_i > 0$, $i = 1, 2, \cdots, n$. Then x solves the problem $NCP(f)$ if and only if there exists a fixed point of the map

$$P(x) = \max\{0, x - \boldsymbol{D} f(x)\}. \tag{10}$$

2.3 A Krawczyk-like Interval Operator

Let $f'([x])$ be an interval extension of $f'(x)$, and I be the $n \times n$ identity matrix. The Krawczyk interval operator is defined by

$$K([x], y, \boldsymbol{D}) = y - \boldsymbol{D} f(y) + (I - \boldsymbol{D} f'([x]))([x] - y), \tag{11}$$

where y is any fixed vector in the interval vector $[x]$. Obviously, Krawczyk interval operator is an interval extension of $x - \boldsymbol{D} f(x)$, i. e.

$$x - \boldsymbol{D} f(x) \in y - \boldsymbol{D} f(y) + (\boldsymbol{I} - \boldsymbol{D} f'([x]))([x] - y) \quad \forall x \in [x]. \tag{12}$$

In [5] Zhenyu Wang and Zuhe Shen introduce a Krawczyk-like interval operator

$$\Gamma([x], y, \boldsymbol{D}) = \max\{0, y - \boldsymbol{D} f(y) + (\boldsymbol{I} - \boldsymbol{D} f'([x]))([x] - y)\}. \tag{13}$$

Theorem 2.4 (Invariant of fixed point) Let $\boldsymbol{D} = \mathrm{diag}(d_1, d_2, \cdots, d_n)$ be a diagonal matrix, where $d_i > 0$, $i = 1, 2, \cdots, n$. And let $y \in [x]$, the interval operator $\Gamma([x], y, \boldsymbol{D})$ be defined as (13), x solves $NCP(f)$. If $x \in [x]$, then

$$x \in \Gamma([x], y, \boldsymbol{D}) \cap [x].$$

Proof From the relations (10) and (12), we have

$$P(x) = \max\{0, x - \boldsymbol{D} f(x)\} \in \Gamma([x], y, \boldsymbol{D}) \quad \forall x \in [x].$$

So

$$x = \max\{0, x - \boldsymbol{D} f(x)\} \in \Gamma([x], y, \boldsymbol{D}).$$

Corollary 2.5 If $\Gamma([x], y, \boldsymbol{D}) \cap [x] = \phi$, then in the rectangle $[x]$ there is no solution to the problem $NCP(f)$.

If $\boldsymbol{A}$ is an interval matrix and $[x]$ is an interval vector, Then

$$w(\boldsymbol{A}([x] - m([x]))) \leqslant \|\boldsymbol{A}\| w([x]).$$

Theorem 2.6 Assume that for any $\boldsymbol{M} \in f'([x])$, $\boldsymbol{M}$ is diagonally dominant and the diagonal part $\boldsymbol{D} = \mathrm{inv}(\mathrm{diag}(d_1, d_2, \cdots, d_n))$ of $f'(m([x]))$ satisfies $d_i > 0$, $1 \leqslant i \leqslant n$. Then $w(\Gamma([x], m([x]), \boldsymbol{D})) \leqslant w([x])$. Furthermore, the strict inequality holds if $\boldsymbol{M}$ is strictly diagonally dominant for any $\boldsymbol{M} \in f'([x])$.

Proof By Theorem 3.4, one has

$$\begin{aligned} w(\Gamma([x], m([x]), \boldsymbol{D}) &\leqslant w(K([x], m([x]), \boldsymbol{D})) \\ &= w((\boldsymbol{I} - \boldsymbol{D} f'([x]))([x] - m([x]))) \leqslant \|\boldsymbol{I} - \boldsymbol{D} f'([x])\| w([x]), \end{aligned}$$

where $K([x], m([x]), \boldsymbol{D})$ is the Krawczyk operator. Let $\boldsymbol{M} = (m_{ij})_{n \times n} \in f'([x])$. since $\boldsymbol{M}$ is diagonally dominant, i. e.

$$\sum_{j \neq i} | m_{ij} | \leqslant | m_{ii} | = d_i, \ i = 1, 2, \cdots, n,$$

one has

$$\| \boldsymbol{I} - \boldsymbol{DM} \| = \max_i \sum_{j \neq i} \left| \frac{m_{ij}}{m_{ii}} \right| \leqslant 1,$$

which indicates that $\| \boldsymbol{I} - \boldsymbol{D} f'([x]) \| \leqslant 1$. So

$$w(\Gamma([x], m([x]), \boldsymbol{D}) < w([x]).$$

Clearly, if the diagonally dominant condition is replaced by the strict diagonally dominant condition, the strict inequality

$$w(\Gamma([x], m([x]), \boldsymbol{D}) < w([x])$$

holds.

Obviously, $f'([x])$ is the first order slope $f[x, z]$ of f.

2.4 Gauss-Seidel Interval Iteration

Krawczyk-Hansen interval iteration is defined by

$$K_i([x]) = y_i - g_i(y) + \sum_{j=1}^{n} \boldsymbol{R}_{ij}([x])([x_j] - y_j), \quad i = 1, 2, \cdots, n,$$

where $g(y) = \boldsymbol{D} f(y)$, $\boldsymbol{R}([x]) = \boldsymbol{I} - \boldsymbol{D} f'([x])$.

Gauss-seidel interval iteration is defined by

$$H_i([x]) = y_i - g_i(y) + \sum_{j=1}^{i-1} \boldsymbol{R}_{ij}([x])(H'_j([x]) - y_j) + \sum_{j=1}^{n} \boldsymbol{R}_{ij}([x])([x_j] - y_j), \quad i = 1, 2, \cdots, n,$$

where $g_i(y) = \boldsymbol{D}_{(i)} f(y_{(i)})$, $i = 1, 2, \cdots, n$; $H'_j([x]) = H_j([x]) \cap [x_j]$, $j = 1, 2, \cdots, i-1$.

Now, we prepare to use Krawczyk-like interval operator and Gauss-Seidel interval iterative method to solve the nonlinear complementarity problem. We introduce an interval iteration

$$\Gamma([x]^{(k)}, y^{(k)}, D^{(k)}) = \max\{0, y^{(k)} - D^{(k)}f(y^{(k)}) + \sum_{j=1}^{i-1} R_{ij}([x]^{(k)})(H'_j([x]) - y_j) + \sum_{j=i}^{n} R_{ij}([x]^{(k)})([x_j] - y_j)\}, \quad i = 1,2,\cdots n,$$

where $y^{(k)}$ and $D^{(k)}$ are chosen as follows

$$y^{(k)} = m([x]^{(k)}),$$
$$D^{(k)} = \mathrm{inv}(\mathrm{diag}(\mathrm{diag}[m(f'([x]^{(k)}))])),$$
$$r_k = \| I - D^{(k)}f'([x]^{(k)}) \|,$$
$$H'_j([x]) = H_j([x]) \cap [x_j], j = 1, 2, \cdots i-1,$$
$$R([x]^{(k)}) = I - D^{(k)}f'([x]^{(k)}).$$

3 Algorithm

If $\Gamma([x], y, D) \cap [x] \neq \emptyset$, using Theorem 2.4, then we can conclude that if there exists a solution x in $[x]$ to the complementarity problem (1), then $x \in \Gamma([x], y, D) \cap [x]$. So, an inclusion iterative method can be derived from the theorem 2.4 to approximate the solutions to the complementarity problem (1).

Algorithm 3.1 Step 1: Select an n-dimensional interval vector $[x]^{(0)}$.

Step 2: compute $[x]^{(k+1)} = \Gamma([x]^{(k)}, y^{(k)}, D^{(k)}) \cap [x]^{(k)}$.

Step 3: If $[x]^{(k+1)} = \emptyset$, then we can conclude that there is no solution to $NCP(f)$ in the interval $[x]^{(k)}$ and terminate the algorithm; If $\| m([x]^{(k)}) - m([x]^{(k-1)}) \| \leqslant 10^{-4}$, then terminate the algorithm.

Otherwise, Let $k = k+1$ and go to Step2.

Clearly, the algorithm will generate an inclusion monotone sequence $[x]^{(k)}$ of interval vectors, i. e.

$$[x]^{(k)} \subseteq [x]^{(k-1)} \quad k = 1, 2, \cdots,$$

so one can conclude that

$$f'([x]^{(k)}) \subseteq f'([x]^{(k-1)}) \quad k = 1, 2, \cdots,$$

since the interval extension $f'([x])$ of $f'(x)$ satisfies the inclusion monotonicity. From Proposition 2.1 to Theorem 2.4, it follows that the solution x to the complementarity problem (1) in $[x]^{(0)}$ is also in all $[x]^{(k)}$ for $k = 1, 2, \cdots$. And we further have the following result for the inclusion method.

Theorem 3.2 Assume that for any $M \in f'([x])$, M is diagonally dominant matrics and the diagonal part $D = \mathrm{inv}(\mathrm{diag}(d_1, d_2, \cdots, d_n))$ of $f'(m([x]))$ satisfies $d_i > 0$, $1 \leqslant i \leqslant n$. And if $[x]^{(k)} \neq \emptyset$, $k = 1, 2, \cdots$, and $r_0 = \| I - D^{(0)}f'([x]^{(0)}) \| < 1$, then there is a unique solution x to the complementarity (1) in $[x]^{(0)}$ and the following hold :

$$x \in [x]^{(k)}, \text{ where } k = 1, 2, \cdots,$$
$$w([x]^{(k)}) \leqslant \Pi_{i=0}^{k-1} r_i \, w([x]^{(0)}).$$

Thus, $[x]^{(k)}$ converges at least linearly to the unique solution x in $[x]^{(0)}$.

Proof From Theorem 2.4

$$x \in \Gamma([x]^{(k)}, y^{(k)}, D^{(k)}) \cap [x]^{(k)}, k = 1, 2, \cdots,$$

so the solution x is in all the interval $[x]^{(k)}$ by Algorithm 3.1, where $k = 1, 2, \cdots$. We notice $r_k = \| I - D^{(k)}f'([x]^{(k)}) \| < 1$, just as the assumptions $r_0 = \| I - D^{(0)}f'([x]^{(0)}) \| < 1$, so the sequence $[x]^{(k)}$ converges to a point interval. Therefore, one can conclude that there is a unique solution to the complementarity problem (1) in the interval $[x]^{(0)}$. It remains to show that the second result holds. From Algorithm 3.1, it follows that

$$w([x]^{(k)}) \leqslant w([x]^{(k-1)}).$$

And from Theorem 2.6, one obtains

$$w(\Gamma([x]^{(k)}, y^{(k)}, D^{(k)})) \leqslant K([x]^{(k)}, y^{(k)}, D^{(k)}) \leqslant r_{k-1} w([x]^{(k-1)}).$$

Since $r_k < 1$, so the result follows.

4 Numerical Experiment

In this section, we give some example to interpret our algorithm.

Example 1 Considering the complementarity problem $NCP(f)$, where

$$f(x)=\begin{pmatrix}2x_1^2-x_1+x_2-1\\ -x_1^2+x_2^2+3x_2+2\end{pmatrix}.$$

Then

$$f'(x)=\begin{pmatrix}4x_1-1 & 1\\ -2x_1 & 2x_2+3\end{pmatrix},\ f'([x])=\begin{pmatrix}4[x]_1-1 & 1\\ -2[x]_1 & 2[x]_2+3\end{pmatrix}.$$

Choose $[x]^{(0)}=\begin{pmatrix}[0.9,\ 1.2]\\ [0,\ 0.2]\end{pmatrix}$, we have $y^{(0)}=(1.05,\ 0.1)^T$,

$$f'(y^{(0)})=\begin{pmatrix}3.2 & 1\\ -2.1 & 3.2\end{pmatrix},\ \boldsymbol{D}^{(0)}=\begin{pmatrix}0.3125 & 0\\ 0 & 0.3125\end{pmatrix},$$

$$[x]^{(1)}=\begin{pmatrix}[0.9109,\ 1.0297]\\ 0\end{pmatrix},\ m([x]^{(1)})=\begin{pmatrix}0.9703\\ 0\end{pmatrix};$$

$$[x]^{(2)}=\begin{pmatrix}[0.9957,\ 1.0055]\\ 0\end{pmatrix},\ m([x]^{(2)})=\begin{pmatrix}1.0006\\ 0\end{pmatrix};$$

$$[x]^{(3)}=[x]^{(4)}=\begin{pmatrix}[1.0000,\ 1.0000]\\ 0\end{pmatrix},\ m([x]^{(4)})=\begin{pmatrix}1.0000\\ 0\end{pmatrix}.$$

$m([x]^{(4)})$ is an approximate solution to the $NCP(f)$.

Example 2 Considering the complementarity problem $LCP(\boldsymbol{M},\ q)$:

$$x\geqslant 0,\ (\boldsymbol{M}x+q)\geqslant 0,\ x^T(\boldsymbol{M}x+q)=0,$$

where

$$\boldsymbol{M}=\begin{pmatrix}2&1&0&0&0\\1&2&1&0&0\\0&1&2&1&0\\0&0&1&2&1\\0&0&0&1&2\end{pmatrix},\ q=\begin{pmatrix}-1\\-1\\-1\\-1\\-1\end{pmatrix}.$$

Choose

$$[x]=\begin{pmatrix}[0,\ 1.0000]\\ [0,\ 0.5000]\\ [0,\ 1.0000]\\ [0,\ 0.5000]\\ [0,\ 1.0000]\end{pmatrix},$$

then

$$[x]^{(1)}=\begin{pmatrix}[0.2500,\ 0.5000]\\ [0,\ 0.3750]\\ [0.0625,\ 0.6250]\\ [0,\ 0.4688]\\ [0.2656,\ 0.6563]\end{pmatrix},\ [x]^{(34)}=\begin{pmatrix}[0.5000,\ 0.5000]\\ [0.0000,\ 0.0000]\\ [0.5000,\ 0.5000]\\ [0,\ 0.0000]\\ [0.5000,\ 0.5000]\end{pmatrix}.$$

$m([x]^{(34)})$ is an approximate solution to the $LCP(\boldsymbol{M},q)$.

Example 3 Considering the complementarity problem $NCP(f)$, where

$$f(x)=\begin{pmatrix} x_1^3-5x_1^2+x_1-2x_2-x_3+5 \\ -x_1+x_2^3+x_2^2-14x_2+2x_3-23 \\ -2x_1-x_2+3x_3^2+4x_3-6 \end{pmatrix}.$$

Choose $[x]^{(0)}=\begin{pmatrix}[3,6]\\ [3,5]\\ [1,4]\end{pmatrix}$, we have $y^{(0)}=(4.5,4,2.5)^T$,

$$f'(y^{(0)})=\begin{pmatrix}23.5 & -2 & -1\\ -1 & 45.0 & 2\\ -2 & -1 & 19.0\end{pmatrix},\ D^{(0)}=\begin{pmatrix}0.0426 & 0 & 0\\ 0 & 0.0222 & 0\\ 0 & 0 & 0.0526\end{pmatrix},$$

$$[x]^{(1)}=\begin{pmatrix}[3.1968,6.0000]\\ [3.2933,4.6444]\\ [1.1019,2.8892]\end{pmatrix},\ m([x]^{(1)})=\begin{pmatrix}4.5984\\ 3.9689\\ 1.9956\end{pmatrix};$$

$$[x]^{(8)}=\begin{pmatrix}[4.9975,5.0025]\\ [3.9996,4.0004]\\ [1.9996,2.0004]\end{pmatrix},\ m([x]^{(8)})=\begin{pmatrix}5.0000\\ 4.0000\\ 2.0000\end{pmatrix};$$

$m([x]^{(8)})$ is an approximate solution to the $NCP(f)$.

Remark 4.1 Above examples illustrate the convergence speed of Algorithm 3.1 is twice as much as that of Krawczyk-Hansen interval algorithm.

Remark 4.2 Choosing a rectangle $[x]$ such that the solution $x\in[x]$ to the problem (1), we must find the solution x by Algorithm 3.1. But if we found out the range of the solution x of the complementarity problem, it would be the best. This will be part of future research.

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