Generalized Drinfel 'd Quantum Double for Weak T-Coalgebras

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Abstract: In this paper, we introduce the notions of a weak Doi-Hopf group module and a group skew pair as a respective generalization of a weak Doi-Hopf module and the usual skew pair. We establish a class of generalized Drinfel 'd doubles which is a class of weak Hopf group coalgebras by a group skew pair.

Key words: group coalgebras, weak T-coalgebras, group skew pairs, generalized Drinfel 'd quantum double

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弱 T-余代数上的广义 Drinfel 'd 量子偶

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[摘要] 在这篇文章中,我们引进了弱 Doi-Hopf 群模和群斜配对等概念,这些是分别作为弱 Doi-Hopf 模和普通斜配对概念的推广. 以此为工具,我们建立了一类广义的 Drinfel 'd 量子偶,这些是一类弱 Hopf 群余代数.

[关键词] 群余代数,弱 T-余代数,群斜配对,广义的 Drinfel 'd 量子偶

In 1990, Drinfel 'd constructed a class of quasitriangular Hopf algebras, called the Drinfeld quantum double, which can give rise to a solution to the well-known Quantum Yang-Baxter Equation (see [4]). In 1991, Majid showed that the categories of the Drinfel 'd quantum double 's representations is equivalent to the categories of Yetter-Drinfel 'd modules (see [6]). Drinfeld 's quantum double has inspired generalizations in several directions (see [7 ~9]). In this paper, we introduce the notions of a weak Doi-Hopf group module and a group skew pair as a respective generalization of a weak Doi-Hopf module in [1] and the usual skew pair in [5]. We establish a class of generalized Drinfel 'd doubles which is a class of weak Hopf group coalgebras by a group skew pair. Finally, we prove that the Yetter-Drinfel 'd modules over a weak Hopf group coalgebra are special cases as weak Doi-Hopf group modules.

1 Generalized Drinfeld 's Double

Definition 1.1 A weak semi-Hopf π-coalgebra $H = \{H_\alpha, m_\alpha, 1_\alpha, \Delta, \varepsilon\}_{\alpha\in\pi}$ is a family of algebras $\{H_\alpha, m_\alpha, 1_\alpha\}_{\alpha\in\pi}$ and at the same time a π-coalgebra $\{H_\alpha, \Delta = \{\Delta_{\alpha,\beta}\}, \varepsilon\}_{\alpha,\beta\in\pi}$, such that:

(i) The comultiplication $\Delta_{\alpha,\beta}: H_{\alpha\beta}\rightarrow H_\alpha\otimes H_\beta$ is a (not necessary unit-preserving) homomorphism of algebras such that

$$(\Delta_{\alpha,\beta}\otimes id_{H_\gamma})\Delta_{\alpha\beta,\gamma}(1_{\alpha\beta\gamma}) = (\Delta_{\alpha,\beta}(1_{\alpha\beta})\otimes 1_\gamma)(1_\alpha\otimes\Delta_{\beta,\gamma}(1_{\beta\gamma})), \tag{1}$$

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$$(\Delta_{\alpha,\beta}\otimes id_{H_\gamma})\Delta_{\alpha\beta,\gamma}(1_{\alpha\beta\gamma}) = (1_\alpha\otimes\Delta_{\beta,\gamma}(1_{\beta\gamma}))(\Delta_{\alpha,\beta}(1_{\alpha\beta})\otimes 1_\gamma), \tag{2}$$

for all $\alpha, \beta, \gamma\in\pi$.

(ii) The counit $\varepsilon: H_1\to k$ is a k-linear map satisfying the identity:

$$\varepsilon(gxh) = \varepsilon(gx_{(2,1)})\varepsilon(x_{(1,1)}h) = \varepsilon(gx_{(1,1)})\varepsilon(x_{(2,1)}h), \tag{3}$$

for all $g, h, x\in H_1$.

Definition 1.2 A weak Hopf π-coalgebra is a weak semi-Hopf π-coalgebra $H = \{H_\alpha, m_\alpha, 1_\alpha, \Delta, \varepsilon\}_{\alpha\in\pi}$ endowed with a family of k-linear maps $S = \{S_\alpha: H_\alpha\to H_{\alpha^{-1}}\}_{\alpha\in\pi}$ (called an antipode) such that the following data hold:

$$m_\alpha(S_{\alpha^{-1}}\otimes id_{H_\alpha})\Delta_{\alpha^{-1},\alpha}(h) = 1_{(1,\alpha)}\varepsilon(h1_{(2,1)}), \tag{4}$$

$$m_\alpha(id_{H_\alpha}\otimes S_{\alpha^{-1}})\Delta_{\alpha,\alpha^{-1}}(h) = \varepsilon(1_{(1,1)}h)1_{(2,\alpha)}, \tag{5}$$

$$S_\alpha(g_{(1,\alpha)})g_{(2,\alpha^{-1})}S_\alpha(g_{(3,\alpha)}) = S_\alpha(g), \tag{6}$$

for all $h\in H_1$, $g\in H_\alpha$ and $\alpha\in\pi$.

Remark $(H_1, m_1, 1_1, \Delta_{1,1}, \varepsilon, S_1)$ is a weak Hopf algebra. The set of axioms of Definition 1.1 is not self-dual. A weak Hopf π-coalgebra H is said to be of finite type if, H_α is finite-dimensional as a k-vector space for all $\alpha\in\pi$. Note that it does not mean that $\oplus_{\alpha\in\pi}H_\alpha$ is finite-dimensional (unless $H_\alpha = 0$ for all but a finite number of $\alpha\in\pi$). The antipode $S = \{S_\alpha\}_{\alpha\in\pi}$ of the weak Hopf π-coalgebra H is said to be bijective if each S_α is bijective.

Definition 1.3 A weak Hopf π-coalgebra $H = \{H_\alpha, m_\alpha, 1_\alpha, \Delta, \varepsilon\}_{\alpha\in\pi}$ is called a weak crossed Hopf π-coalgebra (or, weak T-coalgebar) if it is endowed with a family of algebra isomorphisms $\varphi = \{\varphi_\alpha: H_\beta\to H_{\alpha\beta\alpha^{-1}}\}_{\alpha,\beta\in\pi}$ (the crossing) such that

(i) each φ_α preserves the comultiplication and the counit, i. e., for any $\alpha, \beta, \gamma\in\pi$, we have

$$(\varphi_\alpha\otimes\varphi_\alpha)\Delta_{\beta,\gamma} = \Delta_{\alpha\beta\alpha^{-1},\alpha\gamma\alpha^{-1}}\varphi_\alpha, \tag{7}$$

$$\varepsilon\varphi_\alpha = \varepsilon, \tag{8}$$

(ii) φ is multiplicative in the sense that

$$\varphi_{\alpha\beta} = \varphi_\alpha\varphi_\beta,\ \text{for all } \alpha, \beta\in\pi. \tag{9}$$

Definition 1.4 Let H be a weak T-coalgebra, and let B be a weak T-algebra. A weak π-skew pair is a triple (B, H, σ) (simply, σ) endowed with a family of k-linear maps $\sigma = \{\sigma_{\alpha,\beta}: B_\alpha\otimes H_\beta\to k\}_{\alpha,\beta\in\pi}$ such that the following conditions are satisfied.

- For any $\alpha\in\pi$ and $a\in B_\alpha$, $y\in H_1$,

$$\sigma_{1,1}(1_1, y) = \varepsilon(y)\ \text{and}\ \sigma_{\alpha,1}(a, 1_1) = \varepsilon_\alpha(a). \tag{10}$$

- For any $\alpha, \beta, \gamma\in\pi$ and $a\in B_\alpha$, $x, y\in H_\beta$,

$$\sigma_{\alpha,\beta}(a, xy) = \sigma_{\alpha,\beta}(a_{(1,\alpha)}, y)\sigma_{\alpha,\beta}(a_{(2,\alpha)}, x). \tag{11}$$

- For any $\alpha, \beta, \gamma\in\pi$ and $a\in B_\alpha$, $b\in B_\beta$, $x\in H_{\gamma\delta}$,

$$\sigma_{\alpha\beta,\gamma\delta}(ab, x) = \sigma_{\alpha,\gamma}(a, x_{(1,\gamma)})\sigma_{\beta,\delta}(b, x_{(2,\delta)}). \tag{12}$$

- For any $\alpha, \beta\in\pi$ and $a\in B_\alpha$, $y\in H_\gamma$,

$$\sigma_{\alpha,\gamma}(a, y) = \sigma_{\beta\alpha\beta^{-1},\beta\gamma\beta^{-1}}(\zeta_\beta(a), \phi_\beta(y)). \tag{13}$$

Example 1.5 Let H be a weak T-coalgebra with a bijective antipode S, and let B be a weak T-algebras with an antipode $\mathscr{A}$. Let (B, H, σ) be a weak π-skew pair. Assume that $B = \oplus_\alpha B_\alpha$. We define actions: for any $h\in H_\alpha$, $b\in B_\alpha$,

$$h\rightharpoonup_\alpha b = b_{(2,\alpha)}\sigma_{\alpha,\alpha}(b_{(1,\alpha)}, h),\ b\leftharpoonup_\alpha h = b_{(1,\alpha)}\sigma_{\alpha,\alpha}(b_{(2,\alpha)}, S_\alpha^{-1}S_{\alpha^{-1}}^{-1}(h)),$$

where since every B_α is a coalgebra we can use notation $\Delta(b) = b_{(1,\alpha)}\otimes b_{(2,\alpha)}$ for any $b\in B_\alpha$, with any $\alpha\in\pi$. Then it is not hard to verify that $(B = \oplus_\alpha B_\alpha, \rightarrow, \leftarrow)$ is an H-bimodule π-graded algebra. For example, to check the condition: $\varepsilon_1^t(h)\rightharpoonup_1 1 = h\leftharpoonup_1 1$ for all $h\in H_1$. In fact, we have

$$\sigma_{1,1}(h\rightharpoonup_1 1, l) = \sigma_{1,1}(1_{(2,1)}, l)\sigma_{1,1}(1_{(1,1)}, h) = \sigma_{1,1}(1, lh) = \varepsilon(lh)$$

$$= \varepsilon(l1_{(2,1)})\varepsilon(1_{(1,1)}h) = \sigma_{1,1}(1_{(1,1)}, l1'_{(2,1)})\varepsilon(1'_{(1,1)}h)$$
$$= \sigma_{1,1}(1_{2,1}, l)\sigma_{1,1}(1_{(1,1)}, 1'_{(2,1)})\varepsilon(1'_{(1,1)}h)$$
$$= \sigma_{1,1}(1_{2,1}, l)\sigma_{1,1}(1_{(1,1)}\varepsilon_1^t(h)) = \sigma_{1,1}(\varepsilon_1^t(h) \rightharpoonup_1 1, l)$$

and so $\varepsilon_1^t(h) \rightharpoonup_1 1 = h \rightharpoonup_1 1$.

Similarly, we can prove that $1 \leftharpoonup_1 \varepsilon_1^s(h) = 1 \leftharpoonup_1 h$ for all $h \in H_1$.

On the vector space $H_\alpha \otimes B$, we can define a multiplication by

$$(h\otimes a)(k\otimes b) = \sigma_{\gamma,\gamma}(b_{(1,\gamma)}, \phi_{\alpha^{-1}}(h_{(1,\alpha\gamma\alpha^{-1})}))h_{(2,\alpha)}k \quad ab_{(2,\gamma)}\sigma_{\gamma,\gamma}(b_{(3,\gamma)}, S_\gamma^{-1}(h_{(3,\gamma^{-1})})) \tag{14}$$

for all $h, k \in H_\alpha$, $a \in B_\beta$ and $b \in B_\gamma$. We denote $H_\alpha \otimes B$ with the multiplication given via Eq. (14) by $H_\alpha \quad {}_\sigma B = H_\alpha \quad {}_\sigma \oplus_{\beta\in\pi} B_\beta$. Now, we define $H \quad {}_\sigma B$ with the α-th component as $H_\alpha \quad {}_\sigma \oplus_{\beta\in\pi} B_\beta$, for any $\alpha \in \pi$.

In what follows, we will extend (see [9]).

Let (B, H, σ) be a weak π-skew pair. We write $H \quad {}_\sigma B$ for $H_\alpha \quad {}_\sigma \oplus_\beta B_\beta$.

We have a projection $\bar{J}_\alpha : H_\alpha \quad {}_\sigma \oplus_\beta B_\beta \to \bar{H}_\alpha \quad {}_\sigma \oplus_\beta B_\beta$,

$$J_\alpha(h \quad f) = (1_{\alpha^{-1}} \quad 1)(h \quad f) = (h \quad f)(1_{\alpha^{-1}} \quad 1).$$

Let the αth component of $H \quad {}_\sigma B$, denoted by $(H \quad {}_\sigma B)_\alpha$, be the factor-algebra $\bar{H}_\alpha \quad {}_\sigma \oplus_\beta B_\beta / \mathrm{Ker} J_\alpha$ and let $[h \quad f]$ denote the class of $h \quad f$ in $H \quad {}_\sigma B$. Now we have one of the main results in this paper.

Theorem 1.6 Let (B, H, σ) be a weak π-skew pair. Then $H \quad {}_\sigma B$ is a weak Hopf π-coalgebra with the following structures:

For any $\alpha \in \pi$, αth component $(H \quad {}_\sigma B)_\alpha$ is an associative algebra with the multiplication given in Eq. (14) and with unit $[1_{\alpha^{-1}} \quad 1]$.

The comultiplication is given by

$$\Delta_{\alpha,\beta}([h \quad F]) = [\varphi_\beta(h_{(1,\beta^{-1}\alpha^{-1}\beta)}) \quad F_1] \otimes [h_{(2,\beta^{-1})} \quad F_2] \tag{15}$$

for any $\alpha, \beta \in \pi$, $h \in \bar{H}_{\alpha\beta}$ and $F \in \oplus_{\beta\in\pi} B_\beta$, where we have that $\Delta(F) = F_1 \otimes F_2$.

The counit is obtained by setting

$$\varepsilon([h\otimes f]) = \sigma_{\gamma,\gamma}(f, S_\gamma^{-1}\varepsilon_{\gamma^{-1}}^t(h)) = \sigma_{\gamma,\gamma}(f, 1_{(1,\gamma)})\varepsilon(1_{(2,1)}h) \tag{16}$$

for any $h \in H_1$ and $f \in B_\gamma$, with $\gamma \in \pi$.

For any $\beta \in \pi$, the αth component of the antipode of $H \quad {}_\sigma B$ is given by

$$S_\alpha([h \quad F]) = [\bar{S}_\alpha(h) \quad 1][1_\alpha \quad \mathscr{A}_*(F)] = [\varphi_\alpha S_{\alpha^{-1}}(h) \quad 1][1_\alpha \quad \mathscr{A}_*(F)] \tag{17}$$

for any $h \in \bar{H}_\alpha$ and $F \in \oplus_{\beta\in\pi} B_\beta$, where $\mathscr{A}_*$ is the antipode of B and $\bar{S}_\alpha = \varphi_\alpha \circ S_{\alpha^{-1}}$ is the antipode of $\bar{H}$.

For any $\alpha \in \pi$, the conjugation isomorphism is given by

$$\varphi_\beta([h \quad f]) = [\varphi_\beta(h) \quad \varphi_\beta(f)] \tag{18}$$

for any $h \in \bar{H}_\alpha$ and $f \in B_\gamma$, with $\gamma \in \pi$.

Proof Straightforward.

2 Doi-Hopf π-Modules

Definition 2.1 Let $H = (\{H_\alpha, m_\alpha, 1_\alpha, \Delta, \varepsilon\})$ be a weak Hopf π-coalgebra and let A be an algebra. A right weak π-H-comodule algebra is a π-H-comodulelike object $(A, \rho^A = \{\rho_\lambda^A\}_{\lambda\in\pi})$ such that the following conditions are satisfied.

$$\rho_\alpha^A(ab) = a_{(0,0)}b_{(0,0)} \otimes a_{(1,\alpha)}b_{(1,\alpha)}, \text{ for all } \alpha \in \pi \text{ and } a, b \in A. \tag{19}$$

$$a_{(0,0)} \otimes \varepsilon_\beta^t(a_{(1,1)}) = 1_{(0,0)}a \otimes 1_{(1,\beta)}. \tag{20}$$

Definition 2.2 Let $H = (\{H_\alpha, m_\alpha, 1_\alpha, \Delta, \varepsilon\})$ be a weak Hopf π-coalgebra and let $C = \{C_\alpha, \Delta^C, \varepsilon^C\}$ be a π-coalgebra. C is called a left weak π-H-module coalgebra if there is a family of k-linear maps $\psi^C = \{\psi_\alpha^C : H_\alpha \otimes C_\alpha \to C_\alpha, \alpha \in \pi\}$ such that the following conditions are satisfied.

$$(C_\alpha, \psi_\alpha^C) \text{ is left } H_\alpha\text{-module, for all } \alpha \in \pi. \tag{21}$$

$$\Delta_{\alpha,\beta}^C(h \cdot c) = h_{(1,\alpha)} \cdot c_{(1,\alpha)} \otimes h_{(2,\beta)} \cdot c_{(2,\beta)}, \text{ for all } \alpha, \beta \in \pi, c \in C_{\alpha\beta}, h \in H_{\alpha\beta}. \tag{22}$$

$$\varepsilon(hk\cdot c)=\varepsilon(hk_{(2,1)})\varepsilon(k_{(1,1)}\cdot c) \text{ for all } c\in C_1 \text{ and } h\in H_1. \tag{23}$$

Example 2.3 Let $H=(\{H_\alpha, m_\alpha, 1_\alpha,\}, \Delta, \varepsilon, S)$ be a weak Hopf π-coalgebra. Then (H, m) is a right weak π-H-module coalgebra with $m=\{m_\alpha: H_\alpha\otimes H_\alpha\to H_\alpha\}_{\alpha\in\pi}$.

Definition 2.4 Let $H=(\{H_\alpha, m_\alpha, 1_\alpha,\}, \Delta, \varepsilon, S)$ be a weak Hopf π-coalgebra. Let (A, ρ^A) be a right weak π-H-comodule algebra, and let $(C, \psi^C,)$ be a left weak π-H-module coalgebra. A left-right weak Doi-Hopf π-module M is a left A-module which is also a right π-C-comodulelike object with a comodulelike structure $\rho_\lambda^M: M\to M\otimes C_\lambda$ such that the following compatibility condition holds.

$$\rho_\alpha^M(a\cdot m)=a_{(0,0)}\cdot m_{(0,0)}\otimes a_{(1,\alpha)}\cdot m_{(1,\alpha)}, \tag{24}$$

for any $\alpha, \beta\in\pi$, $m\in M$ and $a\in A$.

Example 2.5 Let H be a weak Hopf π-coalgebra with $H_\alpha=H_{\alpha\lambda}$ and $M=(\{M_\alpha\}, \rho_{\alpha,\beta})_{\alpha,\beta\in\pi}$ a right π-H-comodule with $M_\alpha=M_{\alpha\lambda}$ for a fixed $\alpha\in\pi$ and for all $\lambda\in\pi$. Set $\rho_\lambda^{M_\alpha}=\rho_{\alpha,\lambda}: M_\alpha\to M_\alpha\otimes H_\lambda$ and $\rho_\lambda^{H_\alpha}=\Delta_{\alpha,\lambda}: H_\alpha\to H_\alpha\otimes H_\lambda$. Then $(H_\alpha, \rho_\lambda^{H_\alpha})_{\lambda\in\pi}$ is a right π-H-comodulelike object and M_α is a right Doi-Hopf π-module in the category $\mathscr{M}_{H_\alpha}^{\pi-H}(H)$.

Definition 2.6 Let H be a weak Hopf π-coalgebra with $H_\alpha=H_{\alpha\lambda}$. A left-right weak α-Yetter-Drinfeld module M is a k-space M with a left H-action and a right H-coactionlike object such that the following conditions hold,

$$\rho_\alpha(m)=m_{(0,0)}\otimes m_{(1,\alpha)}\in M\otimes_t H_\alpha=1_{(1,\beta)}\cdot M\otimes 1_{(2,\alpha)}H_\alpha; \tag{25}$$

$$h_{(1,\beta)}\cdot m_{(0,0)}\otimes h_{(2,\alpha)}m_{(1,\alpha)}=(h_{(2,\beta)}\cdot m)_{(0,0)}\otimes(h_{(2,\beta)}\cdot m)_{(1,\alpha)}\phi_{\beta^{-1}}(h_{(1,\beta\alpha\beta^{-1})}) \tag{26}$$

for any $\alpha, \beta\in\pi$, $m\in M$, and $h\in H_{\beta\alpha}$.

Now, we can form the category ${}_H\mathscr{YD}_\alpha^H$ of left-right $\mathscr{YD}_\alpha$-modules which the composition of morphism of $\mathscr{YD}_\alpha$-modules is the standard composition of the underlying linear maps.

Now, we have the following theorem.

Theorem 2.7 Let $H=(\{H_\alpha\}, \Delta, \varepsilon, S)$ be a weak T-coalgebra. Let us fix α in π. Then

(1) H_α can be made into a right weak π-$H^{op}\otimes H$-comodule algebra. The comodulelike structure $\rho^{H\alpha}=\{\rho_\lambda^{H_\alpha}: H_{\alpha\lambda}H_\alpha\otimes(H^{op}\otimes H)_\lambda\}$ is given by the following formula

$$\rho_\lambda^{H_\alpha}(h)=h_{(2,\alpha\lambda)(1,\alpha)}\otimes S_{\lambda^{-1}}^{-1}\phi_{\alpha^{-1}}(h_{(1,\alpha\lambda^{-1}\alpha^{-1})})\otimes h_{(2,\alpha\lambda)(2,\lambda)} \tag{27}$$

for all $h\in H_\alpha$.

(2) H can be turned into a left weak π-$H^{op}\otimes H$module coalgebra. The action of $H^{op}\otimes H$ on H is given by the following formula

$$(h\otimes l)\cdot g=lgh, \text{ for all } g\in H_\alpha, h\in H_\alpha^{op}, l\in H_\alpha.$$

(3) The category $\mathscr{YD}_\alpha(H)$ of the left-right weak α-Yetter-Drinfeld modules and the category ${}_{H\alpha}\mathscr{M}^\pi-H(H^{op}\otimes H)$ of the left-right weak π-Doi-Hopf modules are isomorphic.

Proof (1) it is easy to see that H_α is a right π-$H^{op}\otimes H$-comodulelike object. In what follows, we need to prove that $\rho_\lambda^{H_\alpha}$ is an algebra morphism from $H_\alpha\to H_\alpha\otimes H_\lambda^{op}\otimes H_\lambda$, i.e., Eqs. (19)—(20) hold. In fact, it is easy to see that $\rho_\lambda^{H_\alpha}(1_\alpha)=1_\alpha\otimes 1_\lambda\otimes\lambda$. We also have

$$\begin{aligned}
1_{(0,0)}\otimes\varepsilon_{H^{op}\otimes H}^t(1_{(1,1)}) &= 1_{(2,\alpha)(1,\alpha)}\otimes\tilde{\varepsilon}^t(S^{-1}\phi_{\alpha^{-1}}(1_{(1,1)}))\otimes\varepsilon^t(1_{(2,\alpha)(2,1)})\\
&= 1_{(2,\alpha)}1'_{(1,\alpha)}\otimes\tilde{\varepsilon}^t(S^{-1}\phi_{\alpha^{-1}}(1_{(1,1)}))\otimes\varepsilon^t(1'_{(2,1)})\\
&= 1_{(2,\alpha)}1'_{(1,\alpha)}\otimes S^{-1}\phi_{\alpha^{-1}}(1_{(1,1)})\otimes 1'_{(2,1)})\\
&= 1_{(2,\alpha)(1,\alpha)}\otimes S^{-1}\phi_{\alpha^{-1}}(1_{(1,1)})\otimes 1_{(2,\alpha)(2,1)}=\rho^{H_\alpha}(1_\alpha)
\end{aligned}$$

for $\alpha\in\pi$. As required.

(2), (3) Straightforward.

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新 书 快 报

我校数学与计算机科学学院陈永林教授的专著《广义逆矩阵的理论与方法》,将由南京师范大学出版社于 2005 年 12 月正式出版. 该书系作者积 25 年的研究成果而精心写成,内容丰富,风格独特,习题新颖.

该书共分五章. 第一章主要介绍投影算子的知识;第二章介绍各种常用广义逆,例如 Moore-Penrose 逆、约束逆、Bott-Duffin 逆、广义 Bott-Duffin 逆、Drazin 逆、加权 Drazin 逆、加半正定权广义逆、Eldén 逆等的定义、性质与应用;第三章介绍分块矩阵广义逆的推导方法与其子块独立性,非奇异加边矩阵与 $A_{T,S}^{(2)}$ 的关系;第四章介绍广义逆 $A_{T,S}^{(2)}$ 与约束方程组(矩阵方程)的解的 Cramer 法则;第五章是广义逆 $A_{T,S}^{(2)}$ 的专论,也是常用广义逆的通论,包含了十多个专题:定义方程,算子表示与矩阵表示,极限表示,有限算法,Neumann 型展开式与超幂迭代,基于函数插值的迭代,长方与奇异方程组的半收敛迭代与半收敛迭代矩阵的构造方法,矩阵积的广义逆反序律,连续性与扰动分析. 这章内容是作者的研究成果中最新最重要的部分.