Some Continuous Dependence Results on the Complex Ginzburg-Landau Equation with p-Laplacian

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Abstract: Continuous dependence on a modelling parameter is established for solutions of a problem for a complex Ginzburg-Landau equation with p-Laplacian. We derive a prior estimates that indicate that solutions depend continuously on a parameter in the governing differential equation provided the solutions exist and other proper conditions.

Key words: p-Laplacian, Ginzburg-Landau equation, estimates, dependence

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关于 p-Laplacian 复金兹堡-朗道方程解的连续依赖性

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[摘要] 讨论了关于 p-Laplacian 复金兹堡-朗道方程解对控制系数的连续依赖性问题.通过先验估计,推导出方程的解连续依赖于方程中的某些控制系数.

[关键词] p-Laplacian, 复金兹堡-朗道方程, 先验估计, 连续依赖

0 Introduction

The time-dependent Ginzburg-Landau partial differential equation has been used to model phenomena in a number of different areas in physics and other fields. Recently, Gao and Bu[1] have studied the $n+1$ complex generalized Ginzburg-Landau equation with inhomogeneous Dirichlet boundary problem. Studies have become known as "continuous dependence on modelling" or "structural stability" investigations and have attracted the attentions of a number of researchers (see [2,3]). Yang and Gao[4,5] have discussed the continuous dependence on the modelling parameters for a complex Ginzburg-Landau equation with real and complex coefficients.

In this paper, we will investigate the continuous dependence on parameter for the well-known Ginzburg-Landau equation with p-Laplacian:

$$u_t = au - (\mu + i\nu)\,|u|^2 u + (\alpha + i\beta)\Delta_p u, (x,t) \in \Omega \times (0,T)$$

where $i = \sqrt{-1}$, $1 < p < 2$, the parameters a, ν and β are all real-valued constants with $\mu > 0, \alpha > 0$ and u is a complex-valued scalar function. Although $a, \mu, \nu, \alpha, \beta$ are assumed to be constant, similar results can be obtained if we allow $a, \alpha, \beta, \mu, \nu$ to be functions of the spatial variables with slight modifications. We take them to be constant for more clarity in our analysis. Now by means of Liskevich-Perelmuter inequality in reference [6], we will discuss the continuous dependence results for complex coefficients with p-Laplacian case and we find that

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the solutions depend continuously on a modelling parameter $\mu + i\nu$ provided the solutions exist and $0 < \frac{|\beta|}{\alpha} \leqslant \frac{2\sqrt{p-1}}{|p-2|}$, $|\nu| \leqslant \sqrt{3}\mu$, $u_0 \in L^2(\Omega)$.

1 Continuous Dependence on Parameter $\mu + i\nu$

Now we consider the following problem

$$\begin{cases} u_t = au - (\mu + i|\nu|)|u|^2 u + (\alpha + i\beta)\Delta_p u, & (x,t) \in \Omega \times (0,T), \\ u = u_\alpha, & (x,t) \in \partial\Omega \times (0,T), \\ u(x,0) = u_0(x), & x \in \Omega, \end{cases} \tag{1}$$

where $\mu, \alpha > 0, a, \nu, \beta \in \mathbf{R}, u_\alpha \in H^1(\partial\Omega \times (0,T))$.

Lemma 1.1 (Okazawa and Yokota) Let H is a complex Hilbert space with inner product $< \cdot , \cdot >$ and norm $\| \cdot \|_H$. Let $p \in (1, \infty)$, then for any non-zero $z, w \in H$ with $z \neq w$ we have the following inequality:

$$\frac{|\mathrm{Im} < \|z\|_H^{p-2} z - \|w\|_H^{p-2} w, z - w > |}{\mathrm{Re} < \|z\|_H^{p-2} z - \|w\|_H^{p-2} w, z - w >} \leqslant \frac{|p-2|}{2\sqrt{p-1}}.$$

Suppose we have two solutions u and v that have the same boundary and initial conditions but satisfy the equation with different coefficients $\mu_1 + i\nu_1, \mu_2 + i\nu_2$. Let $w = u - v, \lambda = \mu_1 - \mu_2$, $\tau = \nu_1 - \nu_2$. Then we will find

$$w_t = aw - (\mu_1 + i\nu_1)|u|^2 u + (\mu_2 + i\nu_2)|\nu|^2 v - (\alpha + i\beta)(\Delta_p v - \Delta_p u). \tag{2}$$

To find an priori estimate, we multiply equation (2) by $\bar{w}$ i. e. the conjugate complex function of w and integrate over Ω to obtain

$$\begin{aligned} < w_t, w > &= a\|w\|^2 - < (\mu_1 + i\nu_1)(|u|^2 u - |\nu|^2 v), w > \\ &\quad - < (\lambda + i\tau)|\nu|^2 v, w > - (\alpha + i\beta) < \Delta_p v - \Delta_p u, w >, \end{aligned} \tag{3}$$

where

$$I_1 = (\mu_1 + i\nu_1) < |u|^2 u - |\nu|^2 v, w >, \; I_2 = (\alpha + i\beta) < \Delta_p v - \Delta_p u, w >,$$

and we denote $m = \mathrm{Re} < |u|^2 u - |\nu|^2 v, w >, n = \mathrm{Im} < |u|^2 u - |\nu|^2 v, w >$.

From Lemma 1.1, fixing $p = 4$, then $\frac{|n|}{m} \leqslant \frac{2}{2\sqrt{3}} = \sqrt{3}$, we find that $\mathrm{Re}I_1 = \mu_1 m - \nu_1 n \geqslant 0$ provided $0 < \frac{|\nu_1|}{\mu_1} \leqslant \sqrt{3}$.

From then on, we always suppose $0 < \frac{|\nu_1|}{\mu_1} \leqslant \sqrt{3}$. (we can also see reference [7])

Since

$$\begin{aligned} < \Delta_p v - \Delta_p u, u - v > &= < \mathrm{div}(|\nabla v|^{p-2}\nabla v - |\nabla u|^{p-2}\nabla u), u - v > \\ &= < |\nabla u|^{p-2}\nabla u - |\nabla v|^{p-2}\nabla v, \nabla u - \nabla v > \\ &= a + bi, \end{aligned}$$

where $a = \mathrm{Re} < |\nabla u|^{p-2}\nabla u - |\nabla v|^{p-2}\nabla v, \nabla u - \nabla v >, b = \mathrm{Im} < |\nabla u|^{p-2}\nabla u - |\nabla v|^{p-2}\nabla v, \nabla u - \nabla v >$.

From Lemma 1.1, we get $\frac{|b|}{a} \leqslant \frac{|p-2|}{2\sqrt{p-1}}$, it follows that $\mathrm{Re}I_2 = a\alpha - b\beta \geqslant 0$ provided $0 < \frac{|\beta|}{\alpha} \leqslant \frac{2\sqrt{p-1}}{|p-2|}$.

Taking the real part of (3) we have

$$\begin{aligned} \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|w\|^2 &= a\|w\|^2 - \mathrm{Re}I_1 - \mathrm{Re}\{(\lambda + i\tau) < |v|^2 v, w >\} - \mathrm{Re}I_2 \\ &\leqslant a\|w\|^2 + \sqrt{\lambda^2 + \tau^2}\int_\Omega |v|^3 |w| \,\mathrm{d}x \\ &\leqslant a\|w\|^2 + 3\sqrt{\lambda^2 + \tau^2}\left(\int_\Omega |u|^4 \mathrm{d}x + \int_\Omega |v|^4 \mathrm{d}x\right). \end{aligned}$$

Now set $\eta(t) = \|w\|^2$, so that

$$\frac{\mathrm{d}}{\mathrm{d}t}\eta(t) \leqslant 2a\eta + 6\sqrt{\lambda^2 + \tau^2}\int_\Omega (|u|^4 + |v|^4)\,\mathrm{d}x.$$

Using Gronwall's inequality, we obtain

$$\eta(t) \leqslant 6\sqrt{\lambda^2+\tau^2}e^{2at}\int_0^t\int_\Omega(|u|^4+|v|^4)\mathrm{d}x\mathrm{d}\zeta. \tag{4}$$

Our task is to estimate $\int_0^t\int_\Omega|u|^4\mathrm{d}x\mathrm{d}\zeta$, and it can be accomplished for $\int_0^t\int_\Omega|v|^4\mathrm{d}x\mathrm{d}\zeta$ in the same way.

To this end, we introduce the function $\psi(x,t)$ that satisfies

$$\Delta\psi=0,\ (x,t)\in\Omega\times(0,T),$$
$$\psi=u_\alpha,\ (x,t)\in\partial\Omega\times(0,T).$$

Now we consider the identity

$$\int_0^t\int_\Omega(u-\psi)(\bar{u}_\zeta-a\bar{u}+(\mu_1-i\nu_1)|u|^2\bar{u}-(\alpha-i\beta)\Delta_p\bar{u})\mathrm{d}x\mathrm{d}\zeta=0. \tag{5}$$

Taking the real part of (5) and using Green's identity we have

$$\begin{aligned}\frac{1}{2}\int_\Omega|u|^2\mathrm{d}x+\mu_1\int_0^t\int_\Omega|u|^4\mathrm{d}x\mathrm{d}\zeta &\leqslant \frac{1}{2}\int_\Omega|u_0|^2\mathrm{d}x+|a|\int_0^t\int_\Omega|u|^2\mathrm{d}x\mathrm{d}\zeta\\ &+\int_\Omega|\bar{u}||\psi|\,\mathrm{d}x+\int_\Omega|\bar{u}_0||\psi(x,0)|\,\mathrm{d}x+\int_0^t\int_\Omega|\bar{u}||\psi_\zeta|\,\mathrm{d}x\mathrm{d}\zeta\\ &+|a|\int_0^t\int_\Omega|\bar{u}||\psi|\,\mathrm{d}x\mathrm{d}\zeta+(\mu_1+|\nu_1|)\int_0^t\int_\Omega|u|^3|\psi|\,\mathrm{d}x\mathrm{d}\zeta\\ &+\mathrm{Re}(\alpha-i\beta)\int_0^t\int_\Omega(u-\psi)\Delta_p\bar{u}\mathrm{d}x\mathrm{d}\zeta.\end{aligned} \tag{6}$$

Using Cauchy inequality and Young's inequality (see [8]), we have

$$\int_\Omega|\bar{u}||\psi|\,\mathrm{d}x\leqslant\frac{1}{4}\int_\Omega|u|^2\mathrm{d}x+\int_\Omega|\psi|^2\mathrm{d}x, \tag{7}$$

$$\int_\Omega|\bar{u}_0||\psi(x,0)|\,\mathrm{d}x\leqslant\frac{1}{2}\int_\Omega|u_0|^2\mathrm{d}x+\frac{1}{2}\int_\Omega|\psi(x,0)|^2\mathrm{d}x, \tag{8}$$

$$\int_0^t\int_\Omega|\bar{u}||\psi_\zeta|\,\mathrm{d}x\mathrm{d}\zeta\leqslant\frac{1}{2}\int_0^t\int_\Omega|u|^2\mathrm{d}x\mathrm{d}\zeta+\frac{1}{2}\int_0^t\int_\Omega|\psi_\zeta|^2\mathrm{d}x\mathrm{d}\zeta, \tag{9}$$

$$\int_0^t\int_\Omega|\bar{u}||\psi|\,\mathrm{d}x\mathrm{d}\zeta\leqslant\frac{1}{2}\int_0^t\int_\Omega|u|^2\mathrm{d}x\mathrm{d}\zeta+\frac{1}{2}\int_0^t\int_\Omega|\psi|^2\mathrm{d}x\mathrm{d}\zeta, \tag{10}$$

$$\int_0^t\int_\Omega|u|^3|\psi|\,\mathrm{d}x\mathrm{d}\zeta\leqslant\frac{3}{4}\varepsilon_1\int_0^t\int_\Omega|u|^4\mathrm{d}x\mathrm{d}\zeta+\frac{1}{4}C(\varepsilon_1)\int_0^t\int_\Omega|\psi|^4\mathrm{d}x\mathrm{d}\zeta, \tag{11}$$

$$\begin{aligned}\mathrm{Re}(\alpha-i\beta)\int_0^t\int_\Omega(u-\psi)\Delta_p\bar{u}\mathrm{d}x\mathrm{d}\zeta &= \mathrm{Re}(\alpha-i\beta)\int_0^t\int_\Omega(u-\psi)\mathrm{div}(|\nabla u|^{p-2}\nabla\bar{u})\mathrm{d}x\mathrm{d}\zeta\\ &=-\mathrm{Re}(\alpha-i\beta)\int_0^t\int_\Omega(|\nabla u|^p-|\nabla u|^{p-2}\nabla\bar{u}\nabla\psi)\mathrm{d}x\mathrm{d}\zeta\\ &=-\alpha\int_0^t\int_\Omega|\nabla u|^p\mathrm{d}x\mathrm{d}\zeta+\mathrm{Re}(\alpha-i\beta)\int_0^t\int_\Omega|\nabla u|^{p-2}\nabla\bar{u}\nabla\psi\mathrm{d}x\mathrm{d}\zeta\\ &\leqslant-\alpha\int_0^t\int_\Omega|\nabla u|^p\mathrm{d}x\mathrm{d}\zeta+\sqrt{\alpha^2+\beta^2}\int_0^t\int_\Omega|\nabla u|^{p-1}|\nabla\psi|\mathrm{d}x\mathrm{d}\zeta\\ &\leqslant-\alpha\int_0^t\int_\Omega|\nabla u|^p\mathrm{d}x\mathrm{d}\zeta+\sqrt{\alpha^2+\beta^2}\int_0^t\int_\Omega(\varepsilon_2|\nabla u|^p+C(\varepsilon_2)|\nabla\psi|^2)\mathrm{d}x\mathrm{d}\zeta\\ &=\frac{-\alpha}{2}\int_0^t\int_\Omega|\nabla u|^p\mathrm{d}x\mathrm{d}\zeta+\tilde{C}\int_0^t\int_\Omega|\nabla\psi|^2\mathrm{d}x\mathrm{d}\zeta,\end{aligned} \tag{12}$$

where taking $\varepsilon_2=\dfrac{\alpha}{2\sqrt{\alpha^2+\beta^2}}$, and $1<p<2$.

Substituting (7)—(12) into (6), we can show that there exist positive constant $q,k_1,\cdots,k_6$ such that

$$\begin{aligned}\frac{1}{4}\int_\Omega|u|^2\mathrm{d}x+q\int_0^t\int_\Omega|u|^4\mathrm{d}x\mathrm{d}\zeta &\leqslant\int_\Omega|u_0|^2\mathrm{d}x+k_1\int_0^t\int_\Omega|u|^2\mathrm{d}x\mathrm{d}\zeta\\ &+\frac{1}{2}\int_\Omega|\psi(x,0)|^2\mathrm{d}x+k_2\int_\Omega|\psi|^2\mathrm{d}x+k_3\int_0^t\int_\Omega|\psi_\zeta|^2\mathrm{d}x\mathrm{d}\zeta\end{aligned}$$

$$+ k_4\int_0^t\int_\Omega |\psi|^4 dx d\zeta + k_5\int_0^t\int_\Omega |\psi|^2 dx d\zeta + k_6\int_0^t\int_\Omega |\nabla\psi|^2 dx d\zeta. \tag{13}$$

In order to give an priori estimate for $\int_0^t\int_\Omega |u|^4 dx d\zeta$, we need demonstrate the terms involving ψ are bounded. Now we prepare the following lemma.

Lemma 1.2 (**Rellich identity**)

If the function $\psi(x,t)$ that satisfies

$$\Delta\psi = 0, (x,t) \in \Omega\times(0,T),$$
$$\psi = u_\alpha, \ (x,t) \in \partial\Omega\times(0,T),$$

then

$$\left(\frac{d}{2}-1\right)\int_\Omega |\nabla\psi|^2 dx + \frac{1}{2}\int_{\partial\Omega} x^i n_i \left|\frac{\partial\psi}{\partial n}\right|^2 ds = \frac{1}{2}\int_{\partial\Omega} x^i n_i |\nabla_T\psi|^2 ds - \int_{\partial\Omega} x^i T_i \frac{\partial\psi}{\partial n}\nabla_T\psi ds. \tag{14}$$

where d denote the spatial dimension and the normal and tangential vector to $\partial\Omega$ are n and T, respectively, and $\nabla_T\psi$ is the tangential derivative. It has been proved by Yang and Gao in reference [4]. Here we omit the proof of the Lemma.

(1) If we assume $d>2$, the domain Ω is star shaped with respect to origin and set $\min_{\partial\Omega}|x^i n_i| = K > 0$, then there exist constants c_1, c_2 such that

$$\int_\Omega |\nabla\psi|^2 dx + c_1\int_{\partial\Omega}\left|\frac{\partial\psi}{\partial n}\right|^2 ds \leqslant c_2\int_{\partial\Omega}|\nabla_T\psi|^2 ds, \tag{15}$$

so

$$\int_0^t\int_\Omega |\nabla\psi|^2 dx d\zeta \leqslant c_2\int_0^t\int_{\partial\Omega}|\nabla_T\psi|^2 ds d\zeta. \tag{16}$$

(2) If $d=2$, (14) turns to

$$\int_{\partial\Omega}\left|\frac{\partial\psi}{\partial n}\right|^2 ds \leqslant c_3\int_{\partial\Omega}|\nabla_T\psi|^2 ds. \tag{17}$$

Using Green's identity, we get

$$0 = \int_\Omega \bar\psi\Delta\psi dx = \int_{\partial\Omega}\bar\psi\frac{\partial\psi}{\partial n} ds - \int_\Omega |\nabla\psi|^2 dx. \tag{18}$$

By Cauchy inequality, we have

$$\int_\Omega |\nabla\psi|^2 dx = \int_{\partial\Omega}\bar\psi\frac{\partial\psi}{\partial n} ds \leqslant \frac{1}{2}\int_{\partial\Omega}\left|\frac{\partial\psi}{\partial n}\right|^2 ds + \frac{1}{2}\int_{\partial\Omega}|\psi|^2 ds. \tag{19}$$

Combining (17) with (19), it clear that $\int_\Omega |\nabla\psi|^2 dx$ is bounded even if $d=2$. So for $d \geqslant 2$, $\int_\Omega |\nabla\psi|^2 dx$ and $\int_0^t\int_\Omega |\nabla\psi|^2 dx d\zeta$ is bounded. Using Poincare inequality, we may obtain

$$\int_\Omega |\psi|^2 dx \leqslant c\left(\int_{\partial\Omega}|\psi|^2 ds + \int_\Omega |\nabla\psi|^2 dx\right) \leqslant c_4\int_{\partial\Omega}|\psi|^2 ds + c_5\int_{\partial\Omega}|\nabla_T\psi|^2 ds, \tag{20}$$

it following that

$$\int_0^t\int_\Omega |\psi|^2 dx d\zeta \leqslant c_4\int_0^t\int_{\partial\Omega}|\psi|^2 ds d\zeta + c_5\int_0^t\int_{\partial\Omega}|\nabla_T\psi|^2 ds d\zeta. \tag{21}$$

Using Sobolev embedding theorem, when $d\leqslant 4, H^1(\Omega) \hookrightarrow L^4(\Omega)$, so

$$\int_\Omega |\psi|^4 dx \leqslant c_6\left(\int_{\partial\Omega}|\psi|^2 ds\right)^2 + c_7\left(\int_{\partial\Omega}|\nabla_T\psi|^2 ds\right)^2, \tag{22}$$

from the above inequality, we also have

$$\int_0^t\int_\Omega |\psi|^4 dx d\zeta \leqslant c_6\int_0^t\left(\int_{\partial\Omega}|\psi|^2 ds d\zeta\right)^2 + c_7\int_0^t\left(\int_{\partial\Omega}|\nabla_T\psi|^2 ds d\zeta\right)^2. \tag{23}$$

The last task is to estimate $\int_0^t\int_\Omega |\psi_\zeta|^2 dx d\zeta$, in fact, we can obtain a bound for it in a similar fashion, thus

$$\int_0^t\int_\Omega|\psi_\zeta|^2\mathrm{d}x\mathrm{d}\zeta\leqslant c_8\int_0^t\int_{\partial\Omega}|\psi_\zeta|^2\mathrm{d}s\mathrm{d}\zeta+c_9\int_0^t\int_{\partial\Omega}|\nabla_T\psi_\zeta|^2\mathrm{d}s\mathrm{d}\zeta. \tag{24}$$

From (19) to (24), (13) can be written as

$$\int_\Omega|u|^2\mathrm{d}x\leqslant 4k_1\int_0^t\int_\Omega|u|^2\mathrm{d}x\mathrm{d}\zeta+P(t), \tag{25}$$

where $P(t)$ is bounded. Using Gronwall's inequality, we have

$$\int_0^t\int_\Omega|u|^2\mathrm{d}x\mathrm{d}\zeta\leqslant\int_0^t P(\zeta)\mathrm{e}^{4k_1(t-\zeta)}\mathrm{d}\zeta. \tag{26}$$

Thus, (13) becomes

$$\int_0^t\int_\Omega|u|^4\mathrm{d}x\mathrm{d}\zeta\leqslant\gamma_1\int_0^t P(\zeta)\mathrm{e}^{4k_1(t-\zeta)}\mathrm{d}\zeta+\gamma_2P(t). \tag{27}$$

Similarly,

$$\int_0^t\int_\Omega|\nu|^4\mathrm{d}x\mathrm{d}\zeta\leqslant\gamma_3\int_0^t Q(\zeta)\mathrm{e}^{4l_1(t-\zeta)}\mathrm{d}\zeta+\gamma_4Q(t), \tag{28}$$

where $\gamma_1,\cdots,\gamma_4$ are positive constants and $Q(t)$ is also a term depending only on given data.

Hence combining (27),(28) with (4), it follows that

$$\eta(t)\leqslant 6\sqrt{\lambda^2+\tau^2}\,\mathrm{e}^{2at}\Big(\gamma_1\int_0^t P(\zeta)\mathrm{e}^{4k_1(t-\zeta)}\mathrm{d}\zeta+\gamma_2P(t)+\int_0^t Q(\zeta)\mathrm{e}^{4l_1(t-\zeta)}\mathrm{d}\zeta+\gamma_4Q(t)\Big). \tag{29}$$

Inequality (29) establishes continuous dependence of the solution u of (1) on the modelling parameter $\mu+i\nu$. Hence we obtain the main theorem:

Theorem Let $1<p<2$, u be a solution of problem (1) and Ω is a bounded and star-sharped domain in R^N. If the spatial dimension $2\leqslant d\leqslant 4$, $u_0\in L^2(\Omega)$, $u_\alpha\in H^1(\partial\Omega\times(0,T))$, then u depends continuously on the modelling parameter $\mu+i\nu$ when $0<\frac{|\beta|}{\alpha}\leqslant\frac{2\sqrt{p-1}}{|p-2|}$ and $|\nu|<\sqrt{3}\mu$.

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