Analysis of a Holling IV One-Predator Two-Prey System With Impulsive Effect

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Abstract A Holling type IV Lotka-Volterra one-predator two-prey system with impulsive effect on predator at fixed moments is investigated. Conditions for the stability of the trivial peridic solution and the permanence of the system are established via comparison theorem of impulsive differential equation and analytic methods of differential equation theory.

Key words: predator-prey system, Holling type IV, impulsive effect, extinction, permanence

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具有脉冲效应和 Holling IV 功能性反应的捕食者-食饵系统分析

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[摘要] 构建和分析了在固定时刻脉冲投放捕食者且具有 Holling IV 功能性反应的一个捕食者两个食饵系统, 利用脉冲比较定理和微分方程的分析方法, 得到了平凡周期解稳定和系统持续生存的条件.

[关键词] 捕食者-食饵系统, Holling type IV, 脉冲效应, 灭绝, 持续生存

0 Introduction

Countless organisms live in seasonally or diurnally forced environments. It is necessary and important to consider the models with periodic ecological parameters or perturbations[1]. In fact, almost without exception biological communities are visited by perturbations that occur in a more-or-less periodic fashion. Without exception, in predator-prey system, predator and prey perhaps suffer some perturbations such as fires, floods, etc. That are not suitable to be considered continually. These perturbations bring sudden changes to the system. For example, in integrated pest management (IPM)[2], biological control is one of the most effective methods, which can suppress a pest population by using natural enemies. On occasion, if there are not adequate numbers of natural enemies to provide optimal biological control, the numbers can be increased by releases. Obviously, it is more similar to realize if releases are executed periodically and impulsively. Therefore, it is more significant to introduce

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impulsive differential equations (IDE) into predator-prey system.

Recently the models of predator-prey system with impulsive effect have attracted new attention[3-6]. For example, Zhang et al[3] and Liu and Chen[4] have studied one-predator one-prey system with Holling type-IV[3] and type-II[4] with impulsive perturbations on the predator. Wang et al[5] studied the three food chain with impulsive effects on top predator. Kot et al[6] studied a forced double-Monod model of a predator-prey system in sinusoidally forced inflowing substrate. The predator and prey each exhibit a type-II functional response (Monod Kinetics). In this paper, we will consider the following one-predator two-prey system with Holling type IV functional response and constant periodic release of predator at fixed moments.

$$\begin{cases} \dfrac{dx_1}{dt} = b_1x_1 - x_1^2 - a_1x_1x_2 - \dfrac{m_1x_1y}{1+c_1x_1^2}, \\ \dfrac{dx_2}{dt} = b_2x_2 - x_2^2 - a_2x_1x_2 - \dfrac{m_2x_2y}{1+c_2x_2^2}, \\ \dfrac{dy}{dt} = -b_3y + \dfrac{k_1m_2x_1y}{1+c_1x_1^2} + \dfrac{k_2m_2x_2y}{1+c_2x_2^2}, \quad t \neq n\tau, \\ \Delta x_1 = 0, \quad \Delta x_2 = 0, \quad \Delta y = p, \quad t = n\tau, \end{cases} \tag{1}$$

where $x_i(t)\ (i = 1, 2)$ is population size of prey (pest) species, $y(t)$ is population size of predator species. $b_i > 0\ (i = 1, 2, 3)$ is intrinsic rate of increase or decrease, $a_1 > 0$ and $a_2 > 0$ are parameters representing competitive effects between two prey, $\dfrac{m_1x_1y}{1+c_1x_1^2}$ and $\dfrac{m_2x_2y}{1+c_2x_2^2}$ are the Holling type IV functional responses, $0 < k_1 \leqslant 1$ and $0 < k_2 \leqslant 1$ are the rates of conversing prey into predator. $\Delta y(t) = y(t^+) + y(t)$, τ is the period of the impulse. $p > 0$ is the predators size released each time.

1 Notations and Definitions

Let $\mathbf{R}_+ = [0, \infty)$, $\mathbf{R}_+^3 = \{X \in \mathbf{R}^3 \mid X \geqslant 0\}$. Denote $f = (f_1, f_2, f_3)$ the map defined by the right hand side of the equations of system (1). Let $V: \mathbf{R}_+ \times \mathbf{R}_+^3 \to \mathbf{R}_+$, then V is said to belong to class V_0 if

(1) V is continuous in $(n\tau, (n+1)\tau] \times \mathbf{R}_+^3$ and for each $X \in \mathbf{R}_+^3$, $n \in \mathbf{N}$, there exists

$$\lim_{(t, Y) \to (n\tau^+, X)} V(t, Y) = V(n\tau^+, X).$$

(2) V is locally Lipschitzian in X.

Definition 1 Let $V \in V_0$, then for $(t, X) \in (n\tau, (n+1)\tau] \times \mathbf{R}_+^3$, the upper right derivative of $V(t, X)$ with respect to the impulsive differential system (1) is defined as

$$D^+ V(t, X) = \lim_{h \to 0^+} \sup \frac{1}{h}[V(t+h, X + hf(t, X)) - V(t, X)].$$

The solution of system (1) is a piecewise continuous function $X: \mathbf{R}_+ \to \mathbf{R}_+^3$, $X(t)$ is continuous on $(n\tau, (n+1)\tau]$, $n \in \mathbf{N}$ and $X(n\tau^+) = \lim_{t \to n\tau^+} X(t)$ exists. The smoothness properties of f guarantee the global existence and uniqueness of the solutions of system (1), for the details see book [7].

The following lemma is obvious.

Lemma 1 Let $X(t)$ is a solution of system (1) with $X(0^+) \geqslant 0$, then $X(t) \geqslant 0$ for all $t \geqslant 0$. And further $X(t) > 0$, $t > 0$ if $X(0^+) > 0$.

We will use an important comparison theorem on impulsive differential equation[7].

Lemma 2 Suppose $V \in V_0$. Assume that

$$\begin{cases} D^+ V(t, X) \leqslant g(t, V(t, X)), & t \neq n\tau, \\ V(t, X(t^+)) \leqslant \varphi_n(V(t, X)), & t = n\tau, \end{cases} \tag{2}$$

where $g: \mathbf{R}_+ \times \mathbf{R}_+ \to \mathbf{R}$ is continuous in $(n\tau, (n+1)\tau] \times \mathbf{R}_+$ and for $u \in \mathbf{R}_+$, $n \in \mathbf{N}$, $\lim_{(t, v) \to (n\tau^+, u)} g(t, v) = g(n\tau^+, u)$ exists, $\psi_n: \mathbf{R}_+ \to \mathbf{R}_+$ is non-decreasing. Let $r(t)$ be maximal solution of the scalar impulsive differential equation

$$\begin{aligned} u'(t) &= g(t, u(t)), \quad t \neq n\tau, \\ u(t^+) &= \psi_n(u(t)), \quad t = n\tau, \\ u(0^+) &= u_0 \end{aligned} \tag{3}$$

existing on $[0, \infty)$. Then $V(0^+, X_0) \leqslant u_0$, implies that $V(t, X(t)) \leqslant r(t)$, $t \geqslant 0$, where $X(t)$ is any solution of system (1).

We give some basic properties about the following impulsive equation

$$\begin{cases} y'(t) = -dy(t), & t \neq n\tau, \\ y(t^+) = y(t) + p, & t = n\tau, \\ y(0^+) = y_0, & d > 0. \end{cases} \tag{4}$$

Obviously, system (4) has a unique positive periodic solution

$$\tilde{y}(t) = \frac{p\exp(-d(t-n\tau))}{1-\exp(-d\tau)}, \quad t \in (n\tau, (n+1)\tau], n \in \mathbf{N},$$

$$\tilde{y}(0^+) = \frac{p}{1-\exp(-d\tau)}.$$

For system (4), we have the following results.

Lemma 3 For any solution $y(t)$ of system (4) with $y_0 \geqslant 0$, we have

$$|y(t) - \tilde{y}(t)| \to 0, \quad t \to \infty.$$

For the system (1), we introduce the following two definitions.

Definition 2 The species x_i, $(i = 1, 2)$ and y of system (1) is said to be extinct if $\lim_{t\to\infty} x_i(t) = 0$, $(i = 1, 2)$ and $\lim_{t\to\infty} y(t) = 0$.

Definition 3 The species x_i, $(i = 1, 2)$ and y of system (1) is said to be permanent if there exist positive constants m, M and T_0, such that each positive solution $(x_1(t), x_2(t), y(t))$ of the system satisfies $m \leqslant x_i(t) \leqslant M$, $(i = 1, 2)$ and $m \leqslant y(t) \leqslant M$ for all $t > T_0$. If all species of the system are permanent, then the system is called permanent.

2 Extinction and Permanence

Firstly, we show that all solutions of (1) are uniformly upper bounded.

Theorem 1 There exists a constant $M > 0$, such that $x_1(t) \leqslant M$, $x_2(t) \leqslant M$, $y(t) \leqslant M$ for each solution $X(t) = (x_1(t), x_2(t), y(t))$ of system (1) with all t large enough.

Proof Define function $V(t, X(t))$ such that

$$V(t, X(t)) = k_1x_1(t) + k_2x_2(t) + y(t).$$

then $V \in V_0$. We calculate the upper right derivative of $V(t, X)$ along a solution of system (1) and get the following impulsive differential equation

$$\begin{aligned} D^+V(t) + MV(t) &= k_1(b_1 + M)x_1 - k_1x_1^2 + k_2(b_2+M)x_2 - k_2x_2^2 - (b_3 - M)y, \quad t \neq n\tau, \\ V(t^+) &= V(t) + p, \quad t = n\tau. \end{aligned} \tag{5}$$

Clearly, the right hand side of the first equation in (5) is bounded when $0 < M < b_3$. Select such a M_0 and let M_1 be the bound. Thus (5) leads to

$$\begin{cases} D^+V(t) \leqslant -M_0V(t) + M_1, & t \leqslant n\tau, \\ V(t^+) = V(t) + p, & t = n\tau. \end{cases}$$

According to Lemma 2, we have

$$V(t) \leqslant [V(0^+) - \frac{M_1}{M_0}]\exp(-M_0t) + \frac{p[1-\exp(-nM_0\tau)]\exp(-M_0(t-n\tau))}{1-\exp(-M_0\tau)} + \frac{M_1}{M_0},$$

where $t \in (nT, (n+1)T]$. Hence

$$\lim_{t\to\infty} V(t) \leqslant \frac{M_1}{M_0} + \frac{p\exp(M_0\tau)}{\exp(M_0\tau)-1}.$$

Therefore $V(t, x)$ is ultimately bounded, and we obtain that each positive solution of system (1) is uniformly ultimately bounded. The proof is complete.

Consider the following system

$$\begin{cases}\left.\begin{aligned}\frac{dx_1}{dt} &= b_1x_1 - x_1^2 - \frac{m_1x_1y}{1+c_1x_1^2},\\ \frac{dy}{dt} &= -b_3y + \frac{k_1m_1x_1y}{1+c_1x_1^2},\end{aligned}\right\} & t\neq n\tau,\\ \Delta x_1 = 0,\ \Delta y = p, & t = n\tau.\end{cases} \tag{6}$$

Obviously, system (6) has a trivial periodic solution $(0, \tilde{y}(t))$. We introduce the following theorems from Zhang et al[3].

Lemma 4 For system (6), the trivial periodic solution $(0, \tilde{y}(t))$ is exponentially stable if $\tau < \frac{m_1p}{b_1b_3}$.

Lemma 5 System (6) is permanent if $\tau > \frac{m_1p}{b_1b_3}$.

Theorem 2 For the system (1), the trivial periodic solution $(0, 0, \tilde{y}(t))$ is globally asymptotical stable provided with

$$\tau < \tau_e := \min\left\{\frac{m_1p}{b_1b_3}, \frac{m_2p}{b_2b_3}\right\}.$$

Proof Suppose $(x_1(t), x_2(t), y(t))$ is any solution of (1) with $x_1(0) > 0$, $x_2(0) > 0$ and $y(0) > 0$. By comparing theorem, we may assume that $x_1(t) < b_1$ and $x_2(t) < b_2$, $t \leqslant 0$. By $\tau < \min\left\{\frac{m_1p}{b_1b_3}, \frac{m_2p}{b_2b_3}\right\}$ and Lemma 4, we have that $x_1(t)\to 0$ and $x_2(t)\to 0$ as $t\to\infty$. By Lemma 3, we have $|y(t) - \tilde{y}(t)|\to 0$, $t\to\infty$. The proof is complete.

By the same way, we can prove the following theorem.

Theorem 3 For the system (1), x_1 and y are permanent, $x_2(t)\to 0$ as $t\to\infty$ provided with $\frac{m_1p}{b_1b_3} < \tau < \frac{m_2p}{b_2b_3}$.

Proof Suppose that $(x_1(t), x_2(t), y(t))$ is any solution of (1) with $x_1(0) > 0$, $x_2(0) > 0$ and $y(0) > 0$. By comparing theorem, we may assume that $x_1(t) < b_1$ and $x_2(t) < b_2$, $t\leqslant 0$. By $\tau < \frac{m_2p}{b_2b_3}$ and Lemma 4, we have that $x_2(t)\to 0$ as $t\to\infty$.

Set $\varepsilon_0 > 0$ to be small enough such that $\tau > \frac{m_1p}{(b_1 - m_1\varepsilon_0)(b_3 - k_2m_2\varepsilon_0)}$. For the system (1), there exists $T_1 > 0$ such that $x_2(t) < \varepsilon_0$ for all $t > T_1$. Consider the following comparing system

$$\begin{cases}\left.\begin{aligned}\frac{dx_1}{dt} &\geqslant (b_1 - a_1\varepsilon_0)x_1 - x_1^2 - \frac{m_1x_1y}{1+c_1x_1^2},\\ \frac{dy}{dt} &= -\left(b_3 - \frac{k_2m_2\varepsilon_0}{1+c_2\varepsilon_0^2}\right)y + \frac{k_1m_1x_1y}{1+c_1x_1^2},\end{aligned}\right\} & t\neq n\tau,\\ \Delta x_1 = 0,\ \Delta y = p. & t = n\tau.\end{cases} \tag{7}$$

By Lemma 5, the system (7) is permanence. By comparing theorem we obtain that there exists a $\delta > 0$, such that $\lim_{t\to\infty}\inf x_1(t) \geqslant \delta$. Obviously $\lim_{t\to\infty}\inf y(t) \geqslant \tilde{y}(\tau)$. So x_1 and y are permanent. The proof is complete.

By the same way, we can prove the following theorem.

Theorem 4 For the system (1), x_2 and y are permanent, $x_1(t)\to 0$ as $t\to\infty$ provided with

$\frac{m_2 p}{b_2 b_3} < \tau < \frac{m_1 p}{b_1 b_3}$.

Theorem 5 Denote $B_i := \max\left\{\frac{x_i}{1+c_i x_i}, x_i \in [0, b_i]\right\}, (i = 1, 2)$. Suppose that $b_2 - a_1 b_1 > 0$, $b_1 - a_2 b_2 > 0$, $b_3 - k_1 m_1 B_1 > 0$ and $b_3 - k_2 m_2 B_2 > 0$ hold. System (1) is permanent provided with

$$\tau > \tau_s := \max\left\{\frac{m_2 p}{(b_2 - a_1 b_1)(b_3 - k_1 m_1 B_1)}, \frac{m_1 p}{(b_1 - a_2 b_2)(b_3 - k_2 m_2 B_2)}\right\}. \tag{8}$$

Proof For any solution $(x_1(t), x_2(t), y(t))$ of the system (1), we may suppose that $x_1(0) < b_1$, $x_2(0) < b_2$. We consider the following comparing impulsive equation

$$\begin{cases} \left.\begin{aligned} \frac{dx_1}{dt} &\geqslant (b_1 - a_1 b_2)x_1 - x_1^2 - \frac{m_1 x_1 y}{1 + c_1 x_1}, \\ \frac{dy}{dt} &= -\left(b_3 - \frac{k_2 m_2 b_2}{1 + c_2 b_2}\right)y + \frac{k_1 m_2 x_1 y}{1 + c_1 x_1}, \end{aligned}\right\} & t \neq n\tau \\ \Delta x_1 = 0, \ \Delta y = p. & t = n\tau \end{cases} \tag{9}$$

By Lemma 5, system (9) is permanence if $\tau > \frac{m_1 p}{(b_1 - a_2 b_2)(b_3 - k_2 m_2 B_2)}$. By comparing theorem we obtain that there exists a $\delta_1 > 0$, such that $\lim_{t\to\infty}\inf x_1(t) \geqslant \delta_1$.

By the same way, we can prove that if $\tau > \frac{m_2 p}{(b_2 - a_1 b_1)(b_3 - k_1 m_1 B_1)}$, there exists a $\delta_2 > 0$, such that $\lim_{t\to\infty}\inf x_2(t) \geqslant \delta_2$. Obviously $\lim_{t\to\infty}\inf y(t) \geqslant \tilde{y}(\tau)$. Thus we obtain that x_1, x_2 and y are permanent provided with (8). The proof is complete.

3 Conclusion

In this paper, we investigate the dynamics of a Holling type IV Lotka-Volterra one-predator two-prey system with impulsive effect on predator at fixed moments. We find that if $\tau < \tau_e$, the periodic solution $(0, 0, \tilde{y}(t))$ is globally asymptotically stable; if $\frac{m_1 p}{b_1 b_3} < \tau < \frac{m_2 p}{b_2 b_3}$, x_2 is extinct and x_1 is permanence; if $\frac{m_2 p}{b_2 b_3} < \tau < \frac{m_1 p}{b_1 b_3}$, x_1 is extinct and x_2 is permanence; if $\tau > \tau_s$, the system is permanent.

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