Improvement of Efficiency in FDTD Modeling of Homogeneous Unmagnetized Plasma

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Abstract: The shift operator finite-difference time-domain (SO-FDTD) method is presented for dispersive media. The permittivity of dispersive media is written as rational polynomial function. The relation between ***D*** and ***E*** is derived in time domain. In this paper, the SO-FDTD method is applied to simulate the electromagnetic wave propagation in homogeneous unmagnetized plasma. The accuracy of the method is verified by comparing the solution of ZT-FDTD. The Numerical results also show that the SO-FDTD method is very efficient and its calculation occupies fewer computer resources.

Key words: plasma, electromagnetic wave, FDTD method
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FDTD方法建模均匀非磁化等离子体计算效率的改进

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[摘要] 介绍了色散介质的移位算子时域有限差分方法(SO-FDTD). 通过将色散介质的介电常数写成有理分式函数形式, 推导出时域中电位移矢量 D和电场强度 E之间的关系, 将此方法应用于计算电磁波在均匀非磁化等离子体中的传播. 与 ZT-FDTD方法计算的结果相比, SO-FDTD方法的计算精度得到了证实, 数值结果进一步显示出这一算法计算效率高, 消耗的计算机资源少.

[关键词] 等离子体, 电磁波, FDTD方法

In 1966, Yee first invented finite-difference time-domain method for electromagnetic waves, which is a numerical method directly solve Maxwell's equations in time-domain[1]. It has a lot of advantages, including comparatively easy, effective, needing less CPU times and memory.

Over the last decade, there have been great improvements in using FDTD methods to solve problems of electromagnetism in dispersive media. In 1990, Luebbers et al. published a thesis firstly introducing FDTD method for dispersive media, recursive convolution (RC) method[2]. Then in 1996, in order to improve the accuracy of this calculation, Kelly et al. proposed a new piecewise linear recursive convolution (PLRC) method[3]. Besides, combined with FDTD method, several schemes such as the frequency-dependent Z-transform (ZT)[4], auxiliary differential equation (ADE)[5], the current density convolution (JEC)[6] etc. have been proposed to analysis dispersive media. Among of them, alternating-direction-implicit (ADI) method[7] and shift operator

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(SO) method[8] are frequently applied during the last few years[9-12].

The general constitutive relationship in dispersive medium is in frequency-domain and is transformed to convolution relationship in time-domain. The dielectric constant of dispersive medium is frequency-independent, therefore, it is difficult to apply FDTD directly. In this paper, with introduction of shift operators in dispersive time-domain with the difference approximation method, arithmetic operator intergradations between time-domain and dispersive time-domain are obtained. And the conceptions are concise. The result is acquired without using the Z-transition method. When constitutive relationship in frequency-domain can be written as rational fractional functions, it should be transformed to time-domain, and then to dispersive time-domain. And recursive formulations from $\boldsymbol{D}$ to $\boldsymbol{E}$ can be deduced. The formulations can be used for the FDTD calculation in dispersive medium. With the shift operator method, the related parameters in the electric field are calculated when electromagnetic wave collides with homogeneous unmagnetized plasma. Compared with the ZT solution, the accuracy of results and efficiency of calculation are also obtained.

1 SO-FDTD Method

With collision cold plasma in dispersive medium, the well-known Maxwell's equations and related equations are given as follows:

$$\frac{\partial \boldsymbol{D}}{\partial t} = \nabla \times \boldsymbol{H}, \tag{1}$$

$$\frac{\partial \boldsymbol{H}}{\partial t} = -\frac{1}{\mu_0}\nabla \times \boldsymbol{E}, \tag{2}$$

$$\boldsymbol{D}(\omega) = \varepsilon_0 \varepsilon_r(\omega)\boldsymbol{E}(\omega). \tag{3}$$

As to one-dimension simulation, $\boldsymbol{H}$ can be calculated from $\boldsymbol{E}$, and then $\boldsymbol{D}$ formula can be also calculated by using the FDTD method:

$$\boldsymbol{D}_x^{n+1}(k) = \boldsymbol{D}_x^{k}(k) - \frac{\Delta t}{\Delta Z}\left[\boldsymbol{H}_y^{n+1/2}\left(k+\frac{1}{2}\right) - \boldsymbol{H}_y^{n+1/2}\left(k-\frac{1}{2}\right)\right], \tag{4}$$

$$\boldsymbol{H}_y^{n+1/2}\left(k+\frac{1}{2}\right) = \boldsymbol{H}_y^{n-1/2}\left(k+\frac{1}{2}\right) - \frac{\Delta t}{\mu_0 \Delta Z}\left[\boldsymbol{E}_x^{n}(k+1) - \boldsymbol{E}_x^{n}(k)\right]. \tag{5}$$

Assuming the constitutive relationship formulation (3) in frequency-domain medium, dielectric constant $\varepsilon_r(\omega)$ can be written as rational fractional function:

$$\varepsilon_r(\omega) = \frac{\sum_{n=0}^{N} p_n (j\omega)^n}{\sum_{n=0}^{N} q_n (j\omega)^n}. \tag{6}$$

With transition relationship from frequency-domain to time-domain: $j\omega \rightarrow \partial/\partial t$, insert (6) into (3):

$$\boldsymbol{D}(t) = \varepsilon_0 \varepsilon_r\left(\frac{\partial}{\partial t}\right)\boldsymbol{E}(t). \tag{7}$$

Assuming the function $\left(y(t) = \frac{\partial f(t)}{\partial t}\right)$, its central difference in $(n+0.5)\Delta t$ can approximately be given:

$$\frac{y^{n+1} + y^{n}}{2} = \frac{f^{n+1} - f^{n}}{\Delta t}. \tag{8}$$

Define as

$$z_t f^n = f^{n+1}, \tag{9}$$

z_t is shift operator[8, 12], combine (8) with (9):

$$y^n = \left(\frac{2}{\Delta t}\frac{z_t - 1}{z_t + 1}\right) f^n. \tag{10}$$

After comparison,

$$\frac{\partial}{\partial t}\rightarrow\left(\frac{2}{\Delta t}\frac{z_t-1}{z_t+1}\right).\tag{11}$$

Constitutive relationship in dispersive time-domain can be given:

$$\left[\sum_{n=0}^{N}q_l\left(\frac{2}{\Delta t}\frac{z_t-1}{z_t+1}\right)^l\right]\boldsymbol{D}^n=\varepsilon_0\left[\sum_{n=0}^{N}p_l\left(\frac{2}{\Delta t}\frac{z_t-1}{z_t+1}\right)^l\right]\boldsymbol{E}^n.\tag{12}$$

Suppose $N=2$, and inset it into (12):

$$\left\{\left[q_0+q_1\frac{2}{\Delta t}+q_2\left(\frac{2}{\Delta t}\right)^2\right]z_t^2+\left[2q_0-2q_2\left(\frac{2}{\Delta t}\right)^2\right]z_t+\left[q_0-q_1\frac{2}{\Delta t}+q_2\left(\frac{2}{\Delta t}\right)^2\right]\right\}\boldsymbol{D}^n=$$
$$\left\{\left[p_0+p_1\frac{2}{\Delta t}+p_2\left(\frac{2}{\Delta t}\right)^2\right]z_t^2+\left[2p_0-2p_2\left(\frac{2}{\Delta t}\right)^2\right]z_t+\left[p_0-p_1\frac{2}{\Delta t}+p_2\left(\frac{2}{\Delta t}\right)^2\right]\right\}\varepsilon_0\boldsymbol{E}^n.\tag{13}$$

According to (9), (13) can be written in other form:

$$\boldsymbol{E}^{n+1}=\frac{1}{b_0}\left[a_0\left(\frac{\boldsymbol{D}^{n+1}}{\varepsilon_0}\right)+a_1\left(\frac{\boldsymbol{D}^{n}}{\varepsilon_0}\right)+a_2\left(\frac{\boldsymbol{D}^{n-1}}{\varepsilon_0}\right)-b_1\boldsymbol{E}^n-b_2\boldsymbol{E}^{n-1}\right].\tag{14}$$

Notice that

$$a_0=q_0+q_1\frac{2}{\Delta t}+q_2\left(\frac{2}{\Delta t}\right)^2;\ a_1=2q_0-2q_2\left(\frac{2}{\Delta t}\right)^2;\ a_2=q_0-q_1\frac{2}{\Delta t}+q_2\left(\frac{2}{\Delta t}\right)^2,$$
$$b_0=p_0+p_1\frac{2}{\Delta t}+p_2\left(\frac{2}{\Delta t}\right)^2;\ b_1=2p_0-2p_2\left(\frac{2}{\Delta t}\right)^2;\ b_2=p_0-p_1\frac{2}{\Delta t}+p_2\left(\frac{2}{\Delta t}\right)^2.$$

As to cold plasma, relative dielectric constant is

$$\varepsilon_r(\omega)=1+\frac{\omega_p^2}{\omega(j\nu_c-\omega)}.\tag{15}$$

Where ω_p is plasma frequency, ν_c is average value of electron collision frequency. After comparison, p_0, p_1, p_2, q_0, q_1, q_2 can be calculated with $N=2$: $p_0=\omega_p^2$, $p_1=\nu_c$, $p_2=1$, $q_0=0$, $q_1=\nu_c$, $q_2=1$.

According to (14), $\boldsymbol{E}$ can be calculated. Combine (4) with (5), and the iterative process can be completed.

2 Results and Analysis

We perform a simulation of pulse meeting a unmagnetized plasma. Only the propagation in one-dimensional space is considered. The computational domain is subdivided into 500 cells, and plasma occupies 100 cells. This simulation uses the properties of silver $f_p=28.7$ GHz($\omega_P=2\pi f_p$), $\nu_c=20$ GHz. In the course of this problem, it is necessary to simulate EM waves of 60 GHz. At this frequency, the free space wavelength is $\lambda=3\times10^8/60\times10^9=0.005$ m. Following the rule of the well-known "Courant Condition", a cell size of $\Delta x\leqslant\lambda_{min}/10$ and a time step of $\Delta t=\Delta x/2c_0$ are used, where c_0 is the speed of the light in free space. The incident pulse generated in cell number five is a sine wave with a Gaussian envelop $\left(\sin(\omega t)\exp\left[2\pi\left(\frac{t-t_0}{\tau}\right)^2\right]\right)$. Figs 1 and 2 show the evolution of pulse at different time for two different frequencies.

Figs 1 shows the first simulation at 7 GHz, well below the plasma frequency. Notice that it interacts with the plasma almost as if it is a metal barrier and is almost completely reflected. Figs 2 is a similar simulation at 60 GHz, well above the plasma frequency. A small portion of it is reflected, but the majority of the pulse passes through the plasma.

Furthermore, from Fig 1 and Fig 2, two numerical methods almost have the same accuracy, but there exists a large difference in computing efficiency. Table 1 provides CPU time of this simulation. Due to no Z-transition between the frequency-domain and time-domain, the CPU time of the SO-FDTD can be reduced to almost 1/4~1/5 that of the ZT-FDTD.

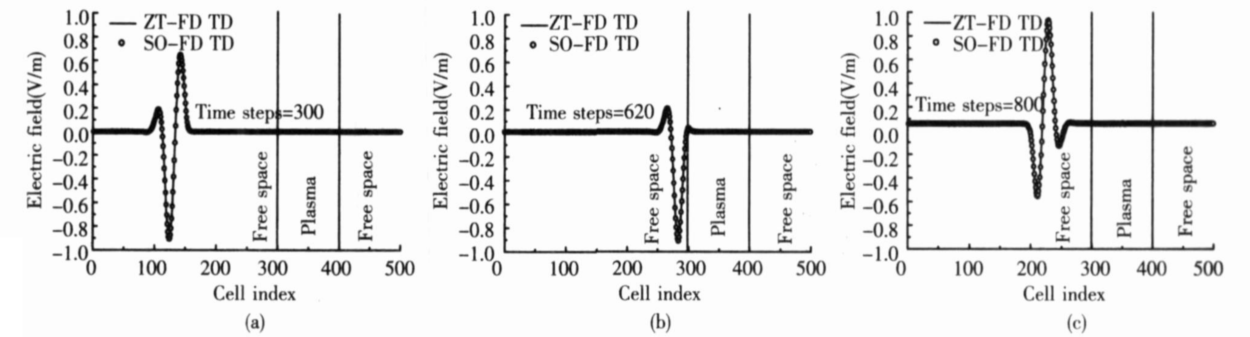

Fig.1 Pulse propagation in free space and striking a plasma medium. The wave has a center frequency of 7 GHz (a)time steps=300; (b) time steps=620; (c)time steps=800

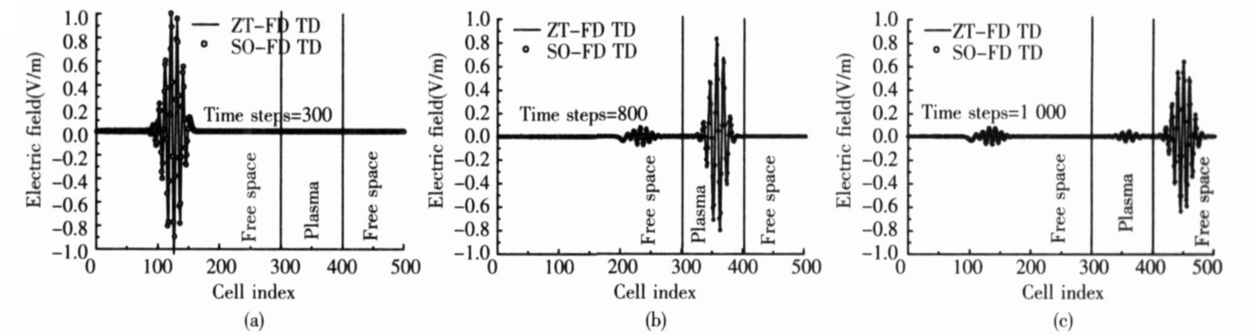

Fig.2 Pulse propagation in free space and colliding a plasma medium. The wave has a center frequency of 60 GHz (a)time steps=300; (b) time steps=800; (c)time steps=1000

Table 1 The CPU time for two numerical methods at different frequencies and time steps

Numerical methods	CPU times/($\times 10^{-2}$ seconds)					
	$f = 7\times 10^9$ Hz			$f = 60\times 10^9$ Hz		
	300	620	800	300	800	1 000
ZT	10.156	18.75	25.00	9.375	26.563	32.813
SO	2.031 3	4.687 5	6.25	3.125	6.25	7.812 5

3 Conclusion

In this paper, the dielectric property of dispersive media is written as rational polynomial function, and the relations between $\boldsymbol{D}$ and $\boldsymbol{E}$ are derived in time-domain. It is named shift operator FDTD (SO-FDTD) method. And with idiographic examples of simulation of electromagnetic wave propagation in free space and colliding plasma medium, the accuracy of ZT-FDTD is verified with the SO-FDTD method. The SO-FDTD method is very efficient and its calculation occupies fewer computer resources, such as CPU time and memory.

The SO-FDTD method has been explained for a 1-D TEM wave in this paper. However, our method could be applied in a two-dimensional or three-dimensional wave.

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