Permanence of a Predator-Prey Model With Impulsive Control Strategy

Tan Yuanshun, Xu Xiaoman

(School of Science, Chongqing Jiaotong University, Chongqing 400074, China)

Abstract: In the present paper, we investigate an impulsive predator-prey model of integrated pest management (IPM) strategy. With the help of qualitative analysis method, small amplitude perturbation skills and comparison theorem we show that when the impulsive period is larger than some critical value, the system can be permanent.

Key words: predator-prey system, permanence, impulsive, IPM

CLC number: O175.1 **Document code:** A **Article ID:** 1001-4616(2012)03-0001-05

一类具脉冲控制策略捕食——食饵模型的持久性

谭远顺,徐晓曼

(重庆交通大学理学院,重庆,400074)

[摘要] 本文考虑了一类脉冲捕食——食饵模型的害虫综合控制策略,应用定性分析方法,小摄动理论和比较定理,得到当脉冲周期大于某一阈值时,系统是持久.

[关键词] 捕食——食饵系统,持久,脉冲,综合害虫控制

In recent decades, it is well know that pest invasion has a significant impact on agriculture, ecology, economic, environment and so on, so it has become more and more popular to control pest by seeking effective strategy. Meanwhile, some advanced and modern methods such as chemical, biological, remote sensing have been adopted accomplishing with the development of society. In this case, the conception of integrated pest management (IPM) comes out, which was introduced in the late 1950s and was widely practised during 1970s and 1980s [1]. IPM is a long-term management strategy that uses a combination of mechanical devices, physical devices, biological, cultural and chemical management to control pests to tolerable levels, with little cost to the grower and minimal effect on beneficial insect, humanity, property and environment [2,3]. It has been proved by experiment that this kind of controlling strategy is more effective than classical one [4].

Models with sudden perturbations describing evolution processes are characterized by the fact that at certain moments of time they abruptly experience a change of state, which are involving in impulsive differential equations. A general amount of the theory of impulsive ordinary differential equations can be found in Bainov and Lakshmikantham [5,6]. Therefore, impulsive differential equations have developed and attracted the interests of many researchers in the last two decades and have been widely applied in almost every domain of applied science in relation to impulsive vaccination, impulsive harvesting and stocking, chemotherapeutic, chemostat, turbidostat

Received date: 2012-04-18.

Foundation item: Supported by the National Science Foundation of China(11271389).

Corresponding author: Tan Yuanshun, doctor, majored in biomathematics. E-mail: tanys@cquc.edu.cn

etc [7,8] .

As everyone knows, most of the mathematical models on the interaction of two species have so far assumed that the response function serves as linear response, Holling I, Holling II, Holling III, Holling VI, Holling-Taylor or Ratio-dependent and so on, while releasing nature enemies and spraying pesticide are co-occurring at the fixed time. As motivated by the above mentioned brief literature, the main purpose of this paper is to establish a mathematical model with Holling Ⅲ S-shaped functional response and investigate the dynamics of such a system.

1 Model Formulation

The standard Lotka-Volterra system has been widely investigated. There has been a mount of excellent results about the effect of ecosystem for periodic change due to season, climate, living conditions, etc [9] . The well known continuous predator-prey model of an ecosystem introduced by Volterra [10] can be formulated as:

$$\begin{cases} y'(t) = \beta b x(t) y(t) - w y(t), \\ x'(t) = x(t)\left(1 - \dfrac{x(t)}{K}\right) - b x(t) y(t), \end{cases} \tag{1}$$

where β, b, w and K are positive constants. $y(t)$ and $x(t)$ represent the densities of predator and prey population, respectively.

Further, we suggest to model such an effect by applying a response function

$$P(x) = \frac{g x^2(t)}{h^2 + x^2(t)}, \tag{2}$$

called the Holling III S-shaped functional response. These coefficients g, h are positive constants. Obviously, the response function (2) is non-monotonic. Simultaneously, on the basis of the above assumptions and IPM strategy, it is more realistic to assume the leasing of nature enemies and the spraying of pesticides should be implemented at different fixed moment. Therefore, considering S-shaped functional response predator-prey system with different fixed moment impulse, the system (1) is rewritten as

$$\begin{cases} \left.\begin{aligned} y'(t) &= \beta b x(t) y(t) - w y(t) \\ x'(t) &= x(t)\left[1 - \dfrac{x(t)}{K}\right] - \dfrac{g x^2(t)}{h^2 + x^2(t)} - b x(t) y(t) \end{aligned}\right\}, & t \neq (n + l - 1)T, t \neq nT, \\ \left.\begin{aligned} y(t) &= -p_2 y(t) \\ x(t) &= -p_1 x(t) \end{aligned}\right\}, & t = (n + l - 1)T, \\ \left.\begin{aligned} y(t) &= \mu \\ x(t) &= 0 \end{aligned}\right\}, & t = nT, \end{cases} \tag{3}$$

where $y(t) = y(t^+) - y(t)$, $x(t) = x(t^+) - x(t)$, $\mu \geqslant 0$ is the released amount of nature enemies at $t = nT$, $0 \leqslant p_2 < 1$ and $0 \leqslant p_1 \leqslant 1$ are the death rate of nature enemies and pests due to spraying pesticides at $t = (n + l - 1)T$, respectively, $n = \{1, 2, \cdots\}$. T is the period of the impulsive effect, and the biological meanings of other coefficients are the same as that of model (1).

2 Analysis of the Model

From reference [11], we know that the positivity and boundedness of the system can be guaranteed, we only prove the permanence of the system here.

If the pest population $x(t)$ is extinction in (3), we give some basic properties of the following subsystem of (3).

$$\begin{cases} y'(t) = -w y(t), & t \neq (n + l - 1)T, t \neq nT, \\ y(t) = -p_2 y(t), & t = (n + l - 1)T, \\ y(t) = \mu, & t = nT, \\ y(0^+) = y_0. \end{cases} \tag{4}$$

System (4) is a periodically linear system, and it is easy to obtain that

$$y^*(t)=\begin{cases}\dfrac{\mu\exp[-w(t-(n-1)T)]}{1-(1-p_2)\exp(-wT)}, & (n-1)T<t\leqslant(n+l-1)T,\\ \dfrac{\mu(1-p_2)\exp[-w(t-(n-1)T)]}{1-(1-p_2)\exp(-wT)}, & (n+l-1)T<t\leqslant nT,\end{cases}$$

with initial value

$$y^*(0^+)=y^*(nT^+)=\frac{\mu}{1-(1-p_2)\exp(-wT)},$$

$$y^*(lT^+)=\frac{\mu(1-p_2)\exp(-wlT)}{1-(1-p_2)\exp(-wT)}$$

is a positive periodic solution of (4). Since the solution of (4) is

$$y(t)=\begin{cases}(1-p_2)^{n-1}\left[y(0^+)-\dfrac{\mu}{1-(1-p_2)\exp(-wT)}\right]\exp(-wt)+y^*(t), \\ \qquad (n-1)T<t\leqslant(n+l-1)T,\\ (1-p_2)^{n}\left[y(0^+)-\dfrac{\mu}{1-(1-p_2)\exp(-wT)}\right]\exp(-wt)+y^*(t), \\ \qquad (n+l-1)T<t\leqslant nT.\end{cases}$$

Lemma 1 System (4) has a positive periodic solution $y^*(t)$ with

$$\int_0^T y^*(t)\,\mathrm{d}t=\frac{\mu[1-p_2\exp(-wlT)+(1-p_2)\exp(-wT)]}{w[1-(1-p_2)\exp(-wT)]}, \tag{5}$$

and for every solution $y(t)$ of (4), we have $|y(t)-y^*(t)|\to 0$ as $t\to\infty$.

Lemma 1 implies that (3) has a pest extinction periodic solution $(y^*(t),0)$. Now, we will discuss the sufficient condition for the permanence of (3) and first give the main theorem.

Theorem 1 System (3) is permanent provided that

$$T-\frac{\mu[1-p_2\exp(-wlT)-(1-p_2)\exp(-wT)]}{w[1-(1-p_2)\exp(-wT)]}>\ln\frac{1}{1-p_1} \tag{6}$$

holds.

Proof Suppose $X(t)=(y(t),x(t))$ is any solution of (3) with $y(0)>0, x(0)>0$. From Lemma 1, we note that $y(t)>y^*(t)-\varepsilon$ for all t large enough. Consequently, $y(t)\geqslant\dfrac{\mu(1-p_2)\exp(-wT)}{1-\exp(-wT)}-\varepsilon\doteq m_2$ for t large enough. Thus we only need to find an $m_1>0$ such that $x(t)\geqslant m_1$. To finish the proof of Theorem 1, we will finish the proof with two steps.

Step I From (6), we can choose $0<m_3<w/(\beta b)$, $\varepsilon_1>0$ small enough such that $\lambda\doteq(1-p_1)\exp\left(\left[1-\left(\dfrac{1}{K}+\dfrac{g}{2h}\right)m_3-\varepsilon\right]T-\dfrac{\mu[1-p_2\exp((-w+\beta bm_3)lT)-(1-p2)\exp((-w+\beta bm_3)t)]}{(w-\beta bm_3)[1-(1-p_2)\exp((-w+\beta bm_3)T)]}\right)>1$. Next, we will prove $x(t)<m_3$ can not hold for all $t\geqslant 0$. Otherwise,

$$\begin{cases}y'(t)\leqslant(-w+\beta bm_3)y(t), & t\neq(n+l-1)T, t\neq nT,\\ y(t)=-p_2y(t), & t=(n+l-1)T,\\ y(t)=\mu, & t=nT,\end{cases}$$

then we derive $y(t)\leqslant y_3(t)$ and $y_3(t)\to y_3^*(t)$ as $t\to\infty$, where $y_3(t)$ is the solution of

$$\begin{cases}y_3(t)=(-w+\beta bm_3)y_3(t), & t\neq(n+l-1)T, t\neq nT,\\ y_3(t)=-p_2y_3(t), & t=(n+l-1)T,\\ y_3(t)=\mu, & t=nT,\\ y_3(0^+)=y_0.\end{cases} \tag{7}$$

and

$$y^*(t)=\begin{cases}\dfrac{\mu\exp[(-w+\beta bm_3)(t-(n-1)T)]}{1-(1-p_2)\exp[(-w+\beta bm_3)T]}, & (n-1)T<t\leqslant(n+l-1)T,\\ \dfrac{\mu(1-p_2)\exp[(-w+\beta bm_3)(t-(n-1)T)]}{1-(1-p_2)\exp[(-w+\beta bm_3)T]}, & (n+l-1)T<t\leqslant nT.\end{cases}$$

Therefore, there exists a $T_1>0$ such that

$$y(t)\leqslant y_3(t)<y_3^*(t)+\varepsilon_1$$

and

$$\begin{cases}x'(t)\geqslant\left[1-\left(\dfrac{1}{K}+\dfrac{g}{2h}\right)m_3-(y_3^*(t)+\varepsilon_1)\right]x(t), & t\neq(n+l-1)T,\\ x(t)=-p_1x(t), & t=(n+l-1)T\end{cases}\tag{8}$$

for $t>T_1$. Choosing a suitable constant $N\in\mathbf{Z}_+$ and $(N+l-1)T\geqslant T_1$. Integrating (8) on $((n+l-1)T,(n+l)T]$, $n\geqslant N$, we get

$$x((n+l)T)\geqslant x((n+l-1)T)(1-p_1)\exp\left(\int_{(n+l-1)T}^{(n+l)T}\left[1-\left(\frac{1}{K}+\frac{b}{2h}\right)m_3-(y^*(t)+\varepsilon_1)\right]\mathrm{d}t\right)=x((n+l-1)T)\lambda.$$

One then obtains that $x((N+n+l-1)T)\geqslant x((N+l)T)\lambda^n\to\infty$ as $n\to\infty$, which is a contradiction to the boundedness of $x(t)$. Thus there exists a $t_1>0$ such that $x_1(t_1)\geqslant m_3$.

Step II If $x(t)\geqslant m_3$ for all $t\geqslant t_1$, then our purpose is obtained. Hence, we need only to consider those solutions which leave the region $\Omega=\{x\in\mathbf{R}_+^2:x_1<m_3\}$ and reenter the region again. Otherwise, $x(t)<m_3$ for some $t\geqslant t_1$. Set $t^*=\inf_{t>t_1}\{x(t)<m_3\}$, there are also two possible case for t^*.

Case I $t^*=(n_1+l-1)T$, $n_1\in\mathbf{Z}_+$. Then we have $x(t)\geqslant m_3$ for $t\in[t_1,t^*]$, and $(1-p_1)\leqslant x(t^{*+})=(x-p_1)x(t^*)<m_3$. Select $n_2,n_3\in\mathbf{Z}_+$ such that

$$(n_2-1)T(-w+\beta bm_3)>\ln\frac{\varepsilon_1}{M+\lambda},(1-p_1)^{n_2}\exp(n_2\lambda_1T)\lambda^{n_3}>1,$$

where $\lambda_1=1-\left(\dfrac{1}{K}+\dfrac{g}{2h}\right)m_3-bM<0$. Denote $T^*=(n_2+n_3)T$, we claim that there must be a $t_2\in(t^*,t^*+T^*]$ such that $x(t_2)>m_3$. Otherwise, consider (7) with $y_3(t^{*+})=y(t^{*+})$, we have

$$y_3(t)=\begin{cases}(1-p_2)^{n-(n_1+1)}\left[y(n_1T^+)-\dfrac{\mu}{1-(1-p_2)\exp[(-w+\beta bm_3)T]}\right]\exp[(-w+\beta bm_3)t]+y^*(t),\\ \qquad (n-1)T<t\leqslant(n+l-1)T,\\ (1-p_2)^{(n-n_1)}\left[y(n_1T^+)-\dfrac{\mu}{1-(1-p_2)\exp[(-w+\beta bm_3)T]}\right]\exp[(-w+\beta bm_3)t]+y^*(t),\\ \qquad (n+l-1)T<t\leqslant nT\end{cases}$$

and $n_1+1\leqslant n\leqslant n_1+n_2+n_3$. Consequently, $|y_3(t)-y_3^*(t)|<(M+\mu)\exp((-w+\beta bm_3)(t-n_1T))<\varepsilon_1$ and $y(t)\leqslant y_3(t)\leqslant y_3^*(t)+\varepsilon_1$ for $t\in[n_1T+(n_2-1)T,t^*+T^*]$, which implies that (8) holds for $t\in[t^*+n_2T,t^*+T^*]$. As in Step I, we get

$$x(t^*+T^*)\geqslant x(t^*+n_2T)\lambda^{n_3}.$$

From (3), we get

$$\begin{cases}x'(t)\geqslant\left[1-\left(\dfrac{1}{K}+\dfrac{g}{2h}\right)m_3-bM\right]x(t), & t\neq(n+l-1)T,\\ x(t)=-p_1x(t), & t=(n+l-1)T\end{cases}\tag{9}$$

for $t\in[t^*,t^*+n_2T]$. Integrating (9) on $[t^*,t^*+n_2T]$, we obtain

$$x(t^*+n_2T)\geqslant m_3(1-p_1)^{n_2}\exp(n_2\lambda_1T).$$

Thus, we have

$$x(t^*+T^*)\geqslant m_3(1-p_1)^{n_2}\exp(n_2\lambda_1T)\lambda^{n_3}>m_3,$$

which is a contradiction.

Let $\tilde{t}=\inf_{t>t^*}\{x(t)>m_3\}$, then for $t\in(t^*,\hat{t})$ we have $x(t)\leqslant m_3$, and $x(\tilde{t})=m_3$. For $t\in(t^*,\tilde{t})$, we get

$$x(t)\geqslant m_3(1-p_1)^{n_2+n_3}\exp[(n_2+n_3)\lambda_1 T].$$

Let $m_1=m_3(1-p_1)^{n_2+n_3}\exp[(n_2+n_3)\lambda_1 T]$, so we have $x(t)\geqslant m_1$ for $t\in(t^*,\tilde{t})$. For $t>\tilde{t}$, the same arguments can be continued since $x(\tilde{t})\geqslant m_3$.

Case II If $t^*\neq(n+l-1)T, n\in\mathbf{Z}_+$, then $x_1(t)\geqslant m_3$ for $t\in[t_1,t^*)$ and $x_1(t^*)=m_3$, suppose $t^*\in((n_1+l-1)T,(n_1+l)T), n_1\in\mathbf{Z}_+$. Then there still have two possible cases for $t\in(t^*,(n_1+l)T)$.

Case II-a If $x(t)\leqslant m_3$ for all $t\in(t^*,(n_1+l)T)$, similarly to Case I we can prove that there exists a $t_2\in[(n_1+l)T,(n_1+l)T+T^*]$ such that $x(t_2)>m_3$. Here we omit it.

Let $\bar{t}=\inf\limits_{t>t^*}\{x(t)>m_3\}$, then $x(t)\geqslant m_3$ for $t\in(t^*,\bar{t})$ and $x(\bar{t})=m_3$. We obtain that

$$x(t)\geqslant m_3(1-p_1)^{n_2+n_3}\exp[(n_2+n_3+1)\lambda_1 T]$$

for $t\in(t^*,\bar{t})$. Let $m_1=m_3(1-p_1)^{n_2+n_3}\exp[(n_2+n_3+1)\lambda_1 T]$, then $x(t)\geqslant m_1$ for $t\in(t^*,\bar{t})$. For $t>\bar{t}$, the same arguments can be continued since $x_1(\bar{t})\geqslant m_3$.

Case II-b If there exists a $t\in(t^*,(n_1+l)T)$ such that $x(t)>m_3$. Let $t'=\inf_{t>t^*}\{x(t)>m_3\}$, then for $t\in(t^*,t')$, we have $x(t)\leqslant m_3$ and $x(t')=m_3$. (9) holds true for $t\in(t^*,t')$, integrating (9) on (t^*,t'), we have

$$x(t)\geqslant x(t^*)\exp[\lambda_1(t-t^*)]\geqslant m_3\exp(\phi T)>m_1.$$

Since $x(t)\geqslant m_3$ for $t^*>t'$, the same arguments can be continued. Hence $x_1(t)\geqslant m_1$ for $t\geqslant t_1$.

Let $\bar{\Omega}=\{m\leqslant y(t),x(t)\leqslant M\}$, where $m=\min\{m_1,m_2\}$, which shows that (3) is permanent. This completes the proof.

[References]

[1] Van den osc R. The Pesticide Conspiracy [M]. Garden City: Doubleday Co, 1978.

[2] Tang S Y, Xiao Y N, Chen L S, et al. Integrated pest management models and their dynamical behaviour [J]. Bulletin of Mathematical Biology, 2005(67): 115-135.

[3] Tan Yuanshun, Chen Lansun. Modelling approach for biological control of insect pest by releasing infected pest [J]. Chaos, Solitons and Fractals, 2009, 39: 304C315.

[4] Van Lenteren J C. Environmental manipulation advantageous to natural enemies of pests [M] // Dent D. Integrated Pest Management. London: Chapman Hall, 1987: 311-320.

[5] Bainov D D, Simeonov P S. Impulsive Differential Equations: Periodic Solutions and Applications [M]. London: Longman, 1993.

[6] Lakmeche A, Arino O. Bifurcation of non trivial periodic solutions of impulsive differential eqauations arising chmotherapeutic treatment [J]. Dyn Continuous Discrete Impulsive Syst, 2000, 7: 265-287.

[7] Liu B, Chen L S. The periodic competing Lotka-Volterra model with impulsive effect [J]. Math Med Biol, 2004, 21: 129-145.

[8] Tang S Y, Chen L S. Density-dependent birth rate, birth pulses and their population dynamic consequences [J]. J Math Biol, 2002, 44: 185-199.

[9] Amine Z, Ortega R. A periodic prey-predator system [J]. J Math Anal Appl, 1994, 185: 477-489.

[10] Volterra V. Variazione e fluttuazini del numerod' individui in specie animali convienti [J]. Mem Acad Nazionale Lincei (Ser. 6), 1926, 2: 31-113.

[11] Tan Yuanshun, Zhang Hong. Dynamics of the oscillative solution for a non-linear ecosystem with impulsive perturbation [J]. Applied Mechanics and Materials, 2012, 5: 471-476.

[责任编辑: 丁　蓉]