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Integrated Pest Management Model with Spraying Pesticides and Releasing Infective Pests

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Abstract:In this paper,a dynamical system of integrated pest management with impulsive birth and impulsive control is investigated. Using the Floquet theorem of impulsive differential equations and the comparison theorem,we obtain the conditions of the globally asymptotical stability of susceptible-pest-eradication periodic solution,and formulate the persistence of the system by numerical simulations. We also discuss the dynamical behaviors of the continuous control system,and obtain the conditions of pest eradication and pest existence as well as the threshold of pest eradication.

Key words:integrated pest management model,impulsive control,global stability,releasing infective pests,spraying pesticides

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基于喷洒杀虫剂及释放病虫的害虫综合管理模型

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[**摘要**] 研究了具有脉冲出生及脉冲控制的害虫综合管理动力系统.利用脉冲微分方程的 Floquet 理论及比较定理,给出了害虫灭绝周期解全局渐近稳定的条件,对系统持续生存情况进行了数值模拟.并讨论连续控制系统的动力学性质,得到了害虫灭绝及持续生存的条件、害虫灭绝的阈值.

[**关键词**] 综合控制害虫模型,脉冲控制,全局稳定性,释放病虫,喷洒杀虫剂

Microbe control has been one of the most important methods for pest control,and using insect viruses is the main method in microbe control. Insect virus has the characters of strong pertinence,being safe to natural enemies and other higher animals,being less polluted to the environment and being harder to create antibody in pests,etc. It is easy to form epidemic in the pest population. Using viruses for pest control aims at making the pests die of sickness,restraining their eating,delaying their growth,decreasing their ability of bearing offspring,and preventing the crops from their harm[1]. Moreover,the viruses can propagate largely in the bowels of adult pests,and discharge with the excrement,which makes the virus diffusion easier. According to this biological feature of viruses,a large amount of adult pests(infective pests)that are infected with viruses by artificial infection are released. In this way, the viruses spread to the pest population through the infective pests,which causes an epidemic among the pests. This special spread method of viruses has successfully received a good effect in some countries,such as Western Samoa,Tonga,Fiji,and so on[2]. Many kinds of insect viruses have already been in experiment in wide area or have

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been produced largely to apply in the pest control of agriculture and forestry[3-5]. And there already have been many successful instances, for example, the use of nucleopolyhedrovirus to control Neodiprion sertifer, Neodiprion lecontei and Diprion hercynia in Canada[1]; the use of nucleopolyhedrovirus to control Heliothis spp in USA[6]; and the use of granulovirus to control vegetable Plutella xylostella in China[7]. There is a vast amount of literature on the applications of microbial diseases to suppress pests[8-11], which mainly focus on biological aspects such as the control mechanism and effect of insect viruses. But there are only a few papers on mathematical models of the dynamics of microbial diseases in pest control[12-13].

Another important method for pest control is chemical control. By spraying pesticides, the pest density will quickly decrease, so the pests can be controlled. Chemical control plays an important role in the history of pest control, but it is acknowledged that a large amount of pesticides will result in environment pollution and damage to human health.

According to the dynamical behaviors of the pest population and the environment, all suitable methods and technologies should be taken together to eradicate the pests or to control the pest density below an economic injury level. That is the viewpoint of integrated pest management[14]. In recent years, the studies of integrated pest management mainly focus on spraying pesticides and releasing natural enemies[15-18]. Only a few focus on using pesticides and insect viruses for integrated pest control.

The rest of the paper is organized as follows. In section 1, based on the integrated pest control strategy of spraying pesticide and releasing infective pests, we discuss the dynamical behaviors of the ordinary system which models the process of continuously spraying pesticide and releasing infective pests. By using the qualitative theory of ordinary differential equations, we obtain the conditions of pest eradication and pest existence as well as the threshold of pest eradication. In section 2, we construct an impulsive system which models the process of periodically birth of susceptible pests at fixed moments, and periodically spray of pesticides and release of infective pests at different fixed time for pest control. Using the Floquet theorem of impulsive differential equations and the comparison theorem, we obtain the conditions of globally asymptotical stability of susceptible-pest-eradication periodic solution, and formulate the persistence of the system by numerical simulations. Section 3 gives the discussions of our results.

1 Continuous Control System

In [13], Goh proposed the following model that uses infective pests to suppress pests

$$\begin{cases}\dot{S}=-rSI,\\ \dot{I}=rSI-wI+u.\end{cases}\tag{1}$$

where S and I denote the density of susceptible and infective pests at time t, respectively. The parameter w represents the death rate of the infective pest population, while r represents its infection rate. The control variable u denotes the release rate of pests that are bred and subsequently infected in a laboratory.

Considering the population growth of the susceptible pests and the integrated pest control strategy of spraying pesticides and releasing infective pests, we improve the model (1). The following assumptions are made.

(A_1) In the absence of the pathogen, the intrinsic birth rate of susceptible pest population is K.

(A_2) Infective pests are incapable of reproducing and cannot attack crops.

(A_3) The parameters μ_1 and μ_2 denote the killing rates of the susceptible and infective pests by spraying pesticide, respectively.

Then the model of differential equations is as the following form:

$$\begin{cases}\dot{S}=KS-\alpha SI-\mu_1 S,\\ \dot{I}=\alpha SI-cI-\mu_2 I+u, \quad S\geqslant 0, I\geqslant 0,\\ S(0)=S_0,\end{cases}\tag{2}$$

where $K>0, \alpha>0, 0\leqslant\mu_1<1, 0\leqslant\mu_2<1, c>0$ and $u>0$. The biological meanings of the coefficients α and c and the control variable u are the same as r, w and μ in the model(1), respectively.

Lemma 1 Every solution of system(2) is bounded in the area $D=\{(S,I)\mid S\geqslant 0, I\geqslant 0\}$.

Lemma 2 (1) System(2) always has a pest-eradication equilibrium$(0,I_1)$, where $I_1=u(c+\mu_2)^{-1}$;

(2) If $R^*=(c+\mu_2)(K-\mu_1)(\alpha u)^{-1}>1$, then system(2) has a unique positive equilibrium(S^*,I^*), where $S^*=(c+\mu_2)\alpha^{-1}-u(K-\mu_1)^{-1}$ and $I^*=(K-\mu_1)\alpha^{-1}$.

Theorem 1 (1) If $R^*<1$, then the pest-eradication equilibrium$(0,I_1)$ is locally asymptotically stable. (2) If $R^*>1$, then the pest-eradication equilibrium$(0,I_1)$ is unstable and the pest-existence equilibrium(S^*,I^*) is locally asymptotically stable.

Proof (1) If $R^*<1$, then the eigenvalues at the equilibrium$(0,I_1)$ are $\lambda_1=-(c+\mu_2)<0$ and $\lambda_2=K-\mu_1-au\times(c+\mu_2)^{-1}<0$. So the equilibrium$(0,I_1)$ is locally asymptotically stable.

(2) If $R^*>1$, then one eigenvalue at the equilibrium$(0,I_1)$ is positive, so$(0,I_1)$ is an unstable saddle point. And both the eigenvalues at the positive equilibrium(S^*,I^*) are positive, so the equilibrium(S^*,I^*) is locally asymptotically stable.

Theorem 2 (1) If $R^*<1$, then the pest-eradication equilibrium$(0,I_1)$ is globally asymptotically stable. (2) If $R^*>1$, then the pest-existence equilibrium(S^*,I^*) is globally asymptotically stable.

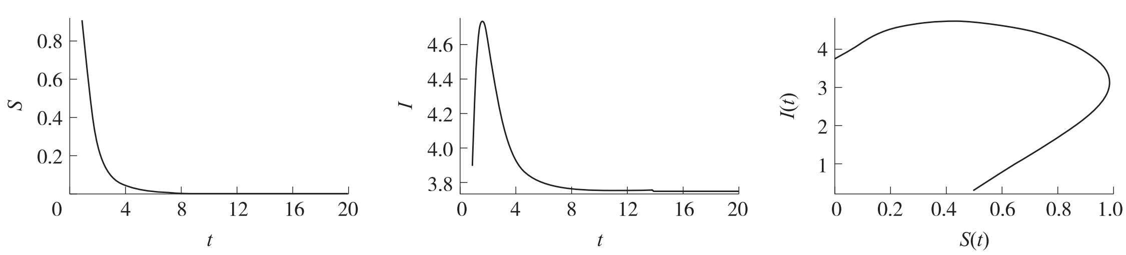
Proof (1) If $R^*<1$, then$(0,I_1)$ is a unique equilibrium of system(2). By Theorem 1, $(0,I_1)$ is locally asymptotically stable. By Lemma 1, all initial values of system(2) are bounded in the area D. Therefore, the pest-eradication equilibrium$(0,I_1)$ is globally asymptotically stable.

(2) If $R^*>1$, select Dulac function $B(S,I)=S^{-1}I^{-1}$ in area $D=\{(S,I)\mid S\geqslant 0, I\geqslant 0\}$, then

$$\frac{\partial(BP)}{\partial S}+\frac{\partial(BQ)}{\partial I}=-\frac{1}{SI^2}\leqslant 0.$$

By Bendixson-Dulac discriminance, we know that there in no limit circle of system(2) in area D. And by Theorem 1, (S^*,I^*) is locally asymptotically stable. From Lemma 1, we know that all initial values of system(2) are bounded in the area D. Therefore, (S^*,I^*) is globally asymptotically stable in area D.

Remark 1 By Theorem 2, if $R^*<1$ (i.e., $u>(c+\mu_2)(K-\mu_1)\alpha^{-1}=u_1^*$), then the boundary equilibrium$(0,I_1)$ of system(2) is globally asymptotically stable, which means that the pests tend to be eradicated; if $R^*>1$ (i.e., $u<u_1^*$), then the positive equilibrium(S^*,I^*) of system(2) is globally asymptotically stable, which means that the susceptible pests exist persistently and coexist with the infective pests at a stable equilibrium. Therefore, $R^*=1$ (i.e., $u=u_1^*$) is the threshold to distinguish pest eradication and pest existence. Fig. 1 is an example of pest eradication, and from it we can see the density of infective pests tends to be $I_1=9.375$, while the density of the susceptible pests tends to be 0. Fig. 2 is a numerical example that shows that the susceptible pests coexist with the infective pests at a stable status, and from it we can see the density of the susceptible pests tends to be $S^*=1.796\ 3$, while the density of infective pests tends to be $I^*=4.5$.

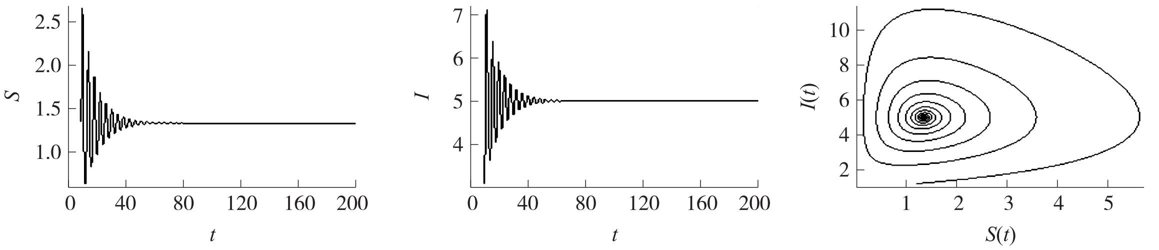


(a)Time series of the susceptible pests　(b)Time series of the infective pests　(c)Phase portrait

Fig. 1 The eradication equilibrium of system(2) with $\mu_1=0.5$, $\mu_2=0.6$, $K=3$, $u=6$, $\alpha=0.8$ and $c=1$

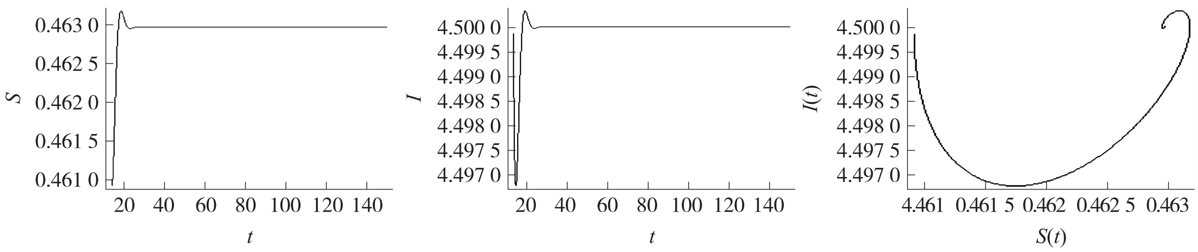
Remark 2 By Theorem 2, we have$(S,I)\to(S^*,I^*)$, as $t\to\infty$. Therefore, in order to keep the density of the pests below a given value E, we only need $S^*<E$, and then we can have $u>((c+\mu_2)\alpha^{-1}-E)(K-\mu_1)=u_2^*$. Then when $u_2^*<u<u_1^*$, the pest will persistently exist and the pest density will be below the given level E. For example,

when $\mu_1=0.3$, $\mu_2=0.3$, $K=3$, $\alpha=0.6$ and $c=1$, we can select $u=1$, then the positive equilibrium of system (2) is (1.796 3, 4.5) (see Fig. 2). When $E=0.5$ is given, we have $u_1^*=5.85$ and $u_2^*=4.5$. So we can select $u=4.6$, then the positive equilibrium of system (2) is (0.463, 4.5) (see Fig. 3). Therefore, the pest density will be below the given level 0.5.



(a)Time series of the susceptible pests (b)Time series of the infective pests (c)Phase portrait

Fig. 2 The persistent-existence equilibrium of system (2) with $\mu_1=0.3$, $\mu_2=0.3$, $K=3$, $u=1$, $\alpha=0.6$ and $c=1$



(a)Time series of the susceptible pests (b)Time series of the infective pests (c)Phase portrait

Fig. 3 The positive equilibrium of system (2) with $\mu_1=0.3$, $\mu_2=0.3$, $K=3$, $u=4.6$, $\alpha=0.6$ and $c=1$

2 Impulsive Control System

In practice, the births of pests, the spraying of pesticides and the releasing of infective pests are not continuous, but periodic. Thus impulsive dynamics system is more practical to describe such phenomenon. In this section, we extend model (2) by considering that the pests are born at a fixed time with a constant period, and that impulsive spraying pesticides and releasing infective pests are done at a fixed time after the births of the pests. That is, we consider the following impulsive differential equations:

$$\begin{cases} \dot{S}=-\alpha SI, & \dot{I}=\alpha SI-cI, \quad t\neq(n-1+\lambda)\tau, t\neq n\tau, \\ \Delta S=KS, & \Delta I=0, \quad t=n\tau, \\ \Delta S=-\mu_1 S, & \Delta I=-\mu_2 I+u, \quad t=(n-1+\lambda)\tau. \end{cases} \tag{3}$$

Where $0\leqslant\lambda<1$, $\Delta S(t)=S(t^+)-S(t)$, $\Delta I(t)=I(t^+)-I(t)$, $h(t^+)=\lim h(u)\,(u\to t)$, $0\leqslant\mu_1<1$ and $0\leqslant\mu_2<1$ respectively denote the death ratios of the susceptible pests and infective pests which are dead due to the use of pesticides at time $t=(n-1+\lambda)\tau$. $u>0$ is the amount of infected pests which are bred in laboratories and released at time $t=(n-1+\lambda)\tau$ in order to drive the susceptible pests towards contracting a disease, which can reduce the susceptible pests population at an acceptably low level to avoid economic loss. $K>0$ is the intrinsic birth rate of susceptible pests at time $t=n\tau$, where $n\in Z^+$, $Z^+=\{1,2,\cdots\}$, and τ is the period of the impulsive effect.

Let $R_+=[0,\infty)$ and $R_+^2=\{z\in R^2 \mid z>0\}$. Denote $f=(f_1,f_2)$ the mapping defined by the right hand of system (3). The solution of system (3), denoted by $z=(S,I): R_+\to R_+^2$, is continuously differentiable on $((n-1)\tau, (n-1+\lambda)\tau)$ and $((n-1+\lambda)\tau, n\tau)$, where $n\in Z^+$ and $0\leqslant\lambda<1$. Obviously, the global existence and uniqueness of the solutions of system (3) is guaranteed by the smoothness of f[19].

Lemma 3 Suppose $z=(S,I)$ is an arbitrary solution of system (3), then we have $z\geqslant0$ for all $t\geqslant0$, when $z(0^+)\geqslant0$. And further we have $z>0$ for $t\geqslant0$, when $z(0^+)>0$.

Consider the following system

$$\begin{cases}\dot{I}=-cI,t\neq n\tau, & t\neq(n-1+\lambda)\tau,\\ \Delta I=-\mu_2 I+u, & t=(n-1+\lambda)\tau,\\ \Delta I=0, & t=n\tau.\end{cases}\tag{4}$$

System(4) has a positive periodic solution

$$\tilde{I}=\begin{cases}\dfrac{u\exp(c(n-2+\lambda)\tau)}{1-(1-\mu_2)\exp(-c\tau)}\exp(-ct), & (n-1)\tau<t\leqslant(n-1+\lambda)\tau,\\ \dfrac{u\exp(c(n-1+\lambda)\tau)}{1-(1-\mu_2)\exp(-c\tau)}\exp(-ct), & (n-1+\lambda)\tau<t\leqslant n\tau.\end{cases}\tag{5}$$

And every solution $I(t)$ of system(4) satisfies $|I(t)-\tilde{I}(t)|\to 0$, as $t\to\infty$.

In system(3), let $S=0$, then we can obtain system(4), so the following Lemma is obvious.

Lemma 4 System(3) has a pest-eradication τ periodic solution $(0,\tilde{I}(t))$.

Theorem 3 (1) If $R_1<1$, then $(0,\tilde{I})$ is locally asymptotically stable. (2) If $R_1>1$, then $(0,\tilde{I})$ is unstable. Where

$$R_1=\frac{c[\exp(c\tau)-1+\mu_2]\ln[(1-\mu_1)(1+K)]}{\alpha u[\exp(c\tau)-1]}.\tag{6}$$

Proof Make the transforms $S=x$ and $I=y+\tilde{I}$, then the approximately linear system of system(3) is

$$\begin{pmatrix}x(t)\\ y(t)\end{pmatrix}=\boldsymbol{\Phi}(t)\begin{pmatrix}x(0)\\ y(0)\end{pmatrix},\ 0\leqslant t<\tau.$$

Where $\boldsymbol{\Phi}$ satisfies

$$\frac{\mathrm{d}\boldsymbol{\Phi}}{\mathrm{d}t}=\begin{pmatrix}-\alpha\tilde{I} & 0\\ * & -c\end{pmatrix}\boldsymbol{\Phi}.$$

And $\boldsymbol{\Phi}(0)=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ is an identity matrix, so the fundamental solution matrix is

$$\boldsymbol{\Phi}(t)=\begin{pmatrix}\exp(\int_0^t(-\alpha\tilde{I}(t))\mathrm{d}s) & 0\\ * & \exp(-ct)\end{pmatrix},\ 0<t<\tau.$$

Accordingly, for the transforms above, the impulsive condition of system(3) is

$$\begin{pmatrix}x((n-1+\lambda)\tau^+)\\ y((n-1+\lambda)\tau^+)\end{pmatrix}=\begin{pmatrix}1-\mu_1 & 0\\ 0 & 1-\mu_2\end{pmatrix}\begin{pmatrix}x((n-1+\lambda)\tau)\\ y((n-1+\lambda)\tau)\end{pmatrix},$$

and

$$\begin{pmatrix}x(n\tau^+)\\ y(n\tau^+)\end{pmatrix}=\begin{pmatrix}1+K & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}x(n\tau)\\ y(n\tau)\end{pmatrix}.$$

The eigenvalues of the single value matrix

$$\boldsymbol{M}=\begin{pmatrix}1-\mu_1 & 0\\ 0 & 1-\mu_2\end{pmatrix}\begin{pmatrix}1+K & 0\\ 0 & 1\end{pmatrix}\boldsymbol{\Phi}(\tau),$$

are

$$\lambda_1=(1-\mu_1)(1+K)\exp\left\{-\frac{u\alpha[\exp(c\tau)-1]}{c[\exp(c\tau)-1+\mu_2]}\right\}\ \text{and}\ \lambda_2=(1-\mu_2)\exp(-(c\tau))<1.$$

According to the Floquet theorem, the sufficient condition on locally asymptotical stability of the pest-eradication periodic solution $(0,\tilde{I})$ is $|\lambda_1|<1$, that is, $R_1<1$. $|\lambda_1|>1$ as $R_1>1$, so $(0,\tilde{I})$ is unstable.

Theorem 4 If $R_1<1$, then $\lim\limits_{t\to\infty}(S(t),I(t))=(0,\tilde{I})$, that is, $(0,\tilde{I})$ is globally attractive.

Proof According to $R_1<1$, we can select $\varepsilon>0$, such that $\delta\triangleq(1-\mu_1)(1+K)\exp[-\int_0^\tau(\tilde{I}-\varepsilon)\mathrm{d}t]$.

In that $\dot{I}>-cI$, we consider the following impulsive system

$$\begin{cases}\dot{y}=-cy, & t\neq n\tau, t\neq(n-1+\lambda)\tau,\\ y(t^+)=y, & t=n\tau,\\ y(t^+)=(1-\mu_2)y+u, & t=(n-1+\lambda)\tau,\\ y(0)=y_0=I_0.\end{cases} \tag{7}$$

According to the comparative theorem of differential equation, we have $I\geqslant y$ and $y\to\tilde{I}$ as $t\to\infty$. Therefore, for t that is large enough, we have $I\geqslant y>\tilde{I}-\varepsilon$.

To be simple, we suppose that the above inequality holds true for all $t>0$. Thus, by system(3) we have

$$\begin{cases}\dot{S}\leqslant-\alpha S(\tilde{I}-\varepsilon), & t\neq n\tau, t\neq(n-1+\lambda)\tau,\\ S(t^+)=(1+K)S, & t=n\tau,\\ S(t^+)=(1-\mu_1)S, & t=(n-1+\lambda)\tau.\end{cases} \tag{8}$$

Thus

$$S[(n+1)\tau]\leqslant S(n\tau^+)\exp[-\alpha\int_{n\tau}^{(n+1)\tau}(\tilde{I}-\varepsilon)\,\mathrm{d}t]=S_0\delta^n\exp[-\alpha\int_{n\tau}^{(n+1)\tau}(\tilde{I}-\varepsilon)\,\mathrm{d}t],$$

$$S[(n+\lambda)\tau]\leqslant S[(n-1+\lambda)\tau^+]\exp[-\alpha\int_{(n-1+\lambda)\tau}^{(n+\lambda)\tau}(\tilde{I}-\varepsilon)\,\mathrm{d}t]=S_0(1-\mu_1)\delta^{n-1}\exp[-\alpha\int_{(n-1)\tau}^{(n-1+\lambda)\tau}(\tilde{I}-\varepsilon)\,\mathrm{d}t]\cdot$$

$$\exp[-\alpha\int_{(n-1+\lambda)\tau}^{(n+\lambda)\tau}(\tilde{I}-\varepsilon)\,\mathrm{d}t]=S_0(1-\mu_1)\delta^{n-1}\exp[-\alpha\int_{(n-1)\tau}^{(n+\lambda)\tau}(\tilde{I}-\varepsilon)\,\mathrm{d}t].$$

Besides, $S[(n+1)\tau]\to0$ and $S[(n+\lambda)\tau]\to0$, as $n\to\infty$. For any $(n+\lambda)\tau<t\leqslant(n+1)\tau$, we have $S[(n+1)\tau]\leqslant S\leqslant S[(n+\lambda)\tau]$, so $S\to0$ as $t\to\infty$.

Because $S\to0$ as $t\to\infty$, so for $0<\varepsilon_1<\dfrac{c}{\alpha}$, there exists a $T_1>0$, such that $0<S<\varepsilon_1$ as $t\geqslant T_1$. By system(3), we have $-cI\leqslant\dot{I}\leqslant(\alpha\varepsilon_1-c)I$, so $y\leqslant I\leqslant z$, and when $t\to\infty$, $y\to\tilde{I}$ and $z\to\tilde{z}$, where $\tilde{I}$ is the solution of system(7), and $\tilde{z}$ is the solution of the following impulsive differential equation

$$\begin{cases}\dot{z}=(\alpha\varepsilon_1-c)z, & t\neq n\tau, t\neq(n-1+\lambda)\tau,\\ z(t^+)=z, & t=n\tau,\\ z(t^+)=(1-\mu_2)z+u, & t=(n-1+\lambda)\tau.\end{cases} \tag{9}$$

And

$$\tilde{z}=\begin{cases}\dfrac{u\exp[-(\alpha\varepsilon_1-c)(n-1+\lambda)\tau]}{\exp(c\tau)-(1-\mu_2)}\exp[(\alpha\varepsilon_1-c)t], & (n-1)\tau<t\leqslant(n-1+\lambda)\tau,\\ \dfrac{u\exp[-(\alpha\varepsilon_1-c)(n+\lambda)\tau]}{\exp(c\tau)-(1-\mu_2)}\exp[(\alpha\varepsilon_1-c)t], & (n-1+\lambda)\tau<t\leqslant n\tau.\end{cases}$$

Therefore, for any $\varepsilon_2>0$, there exists a $T_2>0$, such that $\tilde{I}-\varepsilon_2<I<\tilde{z}+\varepsilon_2$ when $t>T_2$. Let $\varepsilon_1\to0$, then we have $\tilde{I}-\varepsilon_2<I<\tilde{I}+\varepsilon_2$ when t is large enough($\tilde{z}\to\tilde{I}$ as $\varepsilon_1\to0$). The above shows that $I\to\tilde{I}$ when $t\to\infty$. Therefore, $(0,\tilde{I})$ is globally attractive.

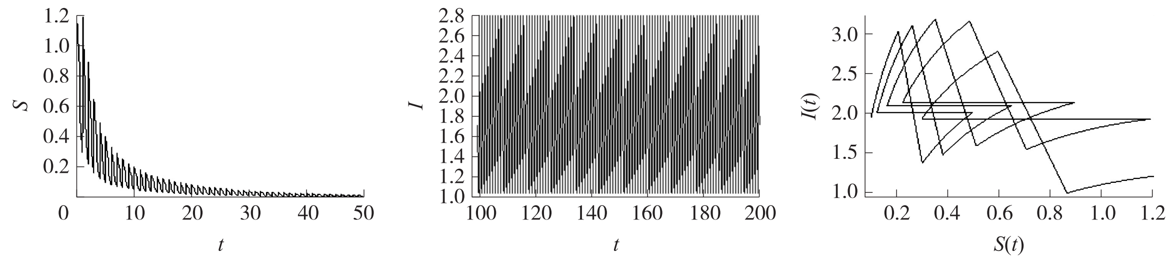
According to Theorem 3 and Theorem 4, we have the following theorem.

Theorem 5 If $R_1<1$, then the pest-eradication periodic solution $(0,\tilde{I})$ of system(3) is globally asymptotically stable.

Remark 3 By Theorem 5, the periodic solution of pest eradication is globally asymptotically stable when $R_1<1$, see Fig. 4. From Fig. 4 we know that $R_1=0.959\,8$ when $\mu_1=0.3$, $\mu_2=0.3$, $K=3$, $u=2.1$, $\alpha=0.6$, $c=1$ and $\tau=1$. Then the density of infective pests $I(t)$ oscillates periodically, while the density of susceptible pests $S(t)$ rapidly tends to 0.

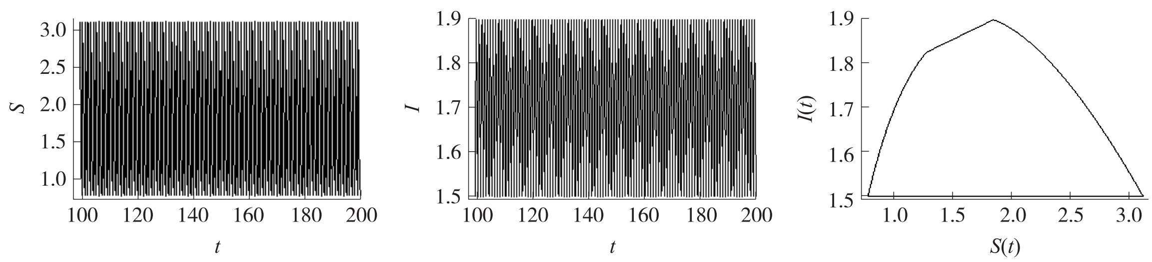
Remark 4 If $R_1>1$, spraying pesticide and releasing infective pest cannot eradicate the pests, and the numerical simulations show that system(3) will persistently exist. Fig. 5 is a numerical example of the persistent existence of the pests. From Fig. 5 we know that $R_1=4.031\,2$ when $\mu_1=0.3$, $\mu_2=0.3$, $K=3$, $u=0.5$, $\alpha=0.6$, $c=1$ and $\tau=1$, and system(3) has a positive periodic solution.

Remark 5 Further, through numerical simulations we have shown that the dynamic behaviors of system(3) change along with the different values of the main parameter u. When $u>c(e^{c\tau}-1+\mu_2)\ln[(1-\mu_2)(1+K)][\alpha(e^{c\tau}-1)]^{-1}=u_3^*$ (that is, $R_1<1$), the pests eradicate (see Fig. 4). When $u<u_3^*$ and it is sufficiently close to u_3^*, system (3) has a positive periodic solution (see Fig. 6(a)). And the smaller the value of u is, the greater the pest density is (see Fig. 6(b)), where $\mu_1=0.3$, $\mu_2=0.3$, $K=3$, $\alpha=0.6$, $c=1$, $\tau=1$ and $u_3^*=2.015\ 64$. When we reduce u to $u=0.01$ and the values of other parameters remain the same as those in Fig. 6, a strange attractor appears in system(3) (see Fig. 7).



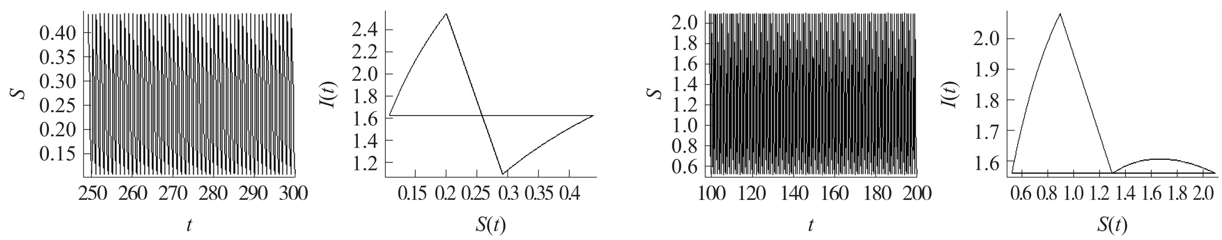
(a)Time series of the susceptible pests　(b)Time series of the infective pests　(c)Phase portrait

Fig. 4 The pest-eradication periodic solution of system(3) with $\mu_1=0.3$, $\mu_2=0.3$, $K=3$, $u=2.1$, $\alpha=0.6$, $c=1$ and $\tau=1$



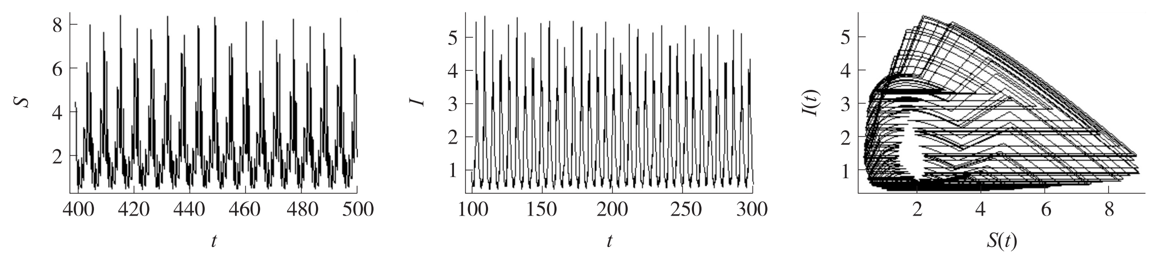
(a)Time series of the susceptible pests　(b)Time series of the infective pests　(c)Phase portrait

Fig. 5 The positive periodic solution of system(3) with $\mu_1=0.3$, $\mu_2=0.3$, $K=3$, $u=0.5$, $\alpha=0.6$, $c=1$ and $\tau=1$



(a) u=1.8　(b) u=1

Fig. 6 Dynamic behaviors of system(3) with $\mu_1=0.3$, $\mu_2=0.3$, $K=3$, $\alpha=0.6$, $c=1$ and $\tau=1$



(a)Time series of the susceptible pests　(b)Time series of the infective pests　(c)Phase portrait

Fig. 7 A strange attractor of system(3) with $\mu_1=0.3$, $\mu_2=0.3$, $K=3$, $u=0.01$, $\alpha=0.6$, $c=1$ and $\tau=1$

3 Discussion

In this paper, based on the integrated pest management strategy of spraying pesticides and releasing infective pests, we constructed both the continuous and impulsive models, and obtained the conditions for pest eradication and pest existence. Further, we got the globally asymptotical stability of the pest-eradication periodic solution. For the continuous model, the threshold for pest eradication is also obtained.

The results show that in the pest control we can select a suitable u according to the price for releasing infective pests and the effect of pesticide, so that the pest will tend to be eradicated or the pest density will be below a given level E.

Indeed, it is impossible to eradicate the pests in reality, and it is not preferable in biology and in economy. Thus we can select an appropriate strategy of integrated pest management to control the pest density to be under the economic threshold. For system(2), when the pesticide is given, we can select the release amount of infective pests that satisfies $u_2^* < u < u_1^*$, so that the pest will persistently exist and the pest density will be below the given level E (see Fig. 3). For system(3), we can select the release amount of infective pests that satisfies $u < u_3^*$, so that the pest will persistently exist and the pest density will be below the given level E. For example, when $\mu_1 = 0.3$, $\mu_2 = 0.3$, $K = 3$, $\alpha = 0.6$ and $c = 1$, we have $u_3^* = 2.015\ 64$. If $E = 0.5$, then we can select $u = 2$, so the pest density will oscillate within a positive amplitude 0.47, that is, the pest density will be below 0.5.

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