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一类具有离散时滞和分布时滞的四元数神经网络同步性研究

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[**摘要**] 本文通过构造合适的 Lyapunov-Krasovskii 函数,利用自由权矩阵方法和四元数不等式技巧,讨论了一类具有离散和分布时滞的四元数神经网络的同步性问题,得出了保证四元数神经网络全局同步性的充分判据.

[**关键词**] 四元数神经网络,Lyapunov-Krasovskii 函数,全局同步性

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Global Synchronization of Quaternion-valued Neural Networks with Discrete and Distributed Delays

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Abstract:In this paper,by constructing suitable Lyapunov-Krasovskii function,using the free weight matrix method and quaternion inequality techniques,the synchronization problem of a class of quaternion neural networks with discrete and distributed delays is discussed, and a sufficient criterion to ensure the global synchronization of quaternion neural networks is obtained.

Key words:quaternion neural networks,Lyapunov-Krasovskii function,global synchronization

自 20 世纪 40 年代以来,由于神经网络在联想记忆、优化、模式识别、故障诊断和信号处理等领域的广泛应用,很多专家和学者对神经网络的同步性、稳定性等问题做了深入的研究. 近几十年来,实值神经网络(RVNNs)和复值神经网络(CVNNs)更是被应用于并行计算、组合优化和量子通信等前沿领域[1-5]. 然而,对于高维数据,如四维信号、人体图像等,RVNNs 和 CVNNs 却无法处理,因此提出并研究四元数神经网络(QVNNs)具有重要的意义[6-10].

尽管 QVNNs 也可以化为 RVNNs 和 CVNNs,但是这种方法的处理效果并不理想,其一,模型维数的增加使得计算变复杂;其二,由于原数据的信号携带了振幅、相位等信息,用多个实值或复值神经网络处理得出的结果,可能会丢失信号所携带的一些信息,而 QVNNs 在处理多维信号时,却可以避免 RVNNs 和 CVNNs 的局限性.

1990 年,Pecora 等对神经网络的同步性做了开创性研究[11],2004 年,文献[12]研究了时滞耦合神经网络的同步性. 2016 年,文献[13]研究了不含有时滞的 CVNNs 的全局同步性. 2018 年,文献[14]考虑了具有 3 种时滞(离散时滞、泄露时滞和分布时滞)的脉冲 CVNNs 的同步性问题.

这些工作在同步性研究中,得到了一些较好的结果,但是对四元数同步性的研究却很少. 对于具有离

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散时滞和分布时滞的 QVNNs 全局同步性研究才处于开始阶段. 在文献[13]中,作者要求模型的激活函数能分离为实部函数和虚部函数. 这种方法会成倍地增加维数,从而导致计算上的困难. 文献[14]中,作者将激活函数看做一个整体来研究系统的同步性,但是只适用于复值神经网络. 于是在此基础上,我们提出了一类具有离散时滞和分布时滞的四元数神经网络模型,在研究过程中将激活函数看作一个整体,以此方法来研究系统的全局同步性.

1　预备知识

1.1　四元数基本知识

四元数,可以写成以下形式:

$$q=q_0+q_1\boldsymbol{i}+q_2\boldsymbol{j}+q_3\boldsymbol{k}\in H,$$

式中,q_0,q_1,q_2,q_3 为实系数,且包含一个实部(用 $R(q)=q_0$ 表示)和 3 个虚部(用 $\boldsymbol{J}(q)=q_1\boldsymbol{i}+q_2\boldsymbol{j}+q_3\boldsymbol{k}$ 表示). 虚数单位 $\boldsymbol{i},\boldsymbol{j},\boldsymbol{k}$ 遵循以下原则:

$$\begin{cases}\boldsymbol{i}^2=\boldsymbol{j}^2=\boldsymbol{k}^2=-1;\\ \boldsymbol{ij}=-\boldsymbol{ji}=\boldsymbol{k},\ \boldsymbol{jk}=-\boldsymbol{kj}=\boldsymbol{i},\boldsymbol{ki}=-\boldsymbol{ik}=\boldsymbol{j}.\end{cases}$$

对于四元数 $q=q_0+q_1\boldsymbol{i}+q_2\boldsymbol{j}+q_3\boldsymbol{k}\in H$ 和 $p=p_0+p_1\boldsymbol{i}+p_2\boldsymbol{j}+p_3\boldsymbol{k}\in H$,满足

①加法:$p+q=(p_0+q_0)+(p_1+q_1)\boldsymbol{i}+(p_2+q_2)\boldsymbol{j}+(p_3+q_3)\boldsymbol{k}$;

②乘法:$pq=(p_0q_0-p_1q_1-p_2q_2-p_3q_3)+(p_0q_1+p_1q_0+p_2q_3-p_3q_2)\boldsymbol{i}+(p_0q_2+p_2q_0-p_1q_3+p_3q_1)\boldsymbol{j}+(p_0q_3+p_3q_0+p_1q_2-p_2q_1)\boldsymbol{k}$;

③q 的共轭 q^*:$q^*=q_0-q_1\boldsymbol{i}-q_2\boldsymbol{j}-q_3\boldsymbol{k}$;

④q 的模 $|q|$:$|q|=\sqrt{qq^*}=\sqrt{q_0^2+q_1^2+q_2^2+q_3^2}$;

⑤令 $\boldsymbol{u}=(u_1,u_2,\cdots,u_n)^{\mathrm{T}}\in H^n$,则 $\boldsymbol{u}$ 的模为 $|\boldsymbol{u}|=(|u_1|,|u_2|,\cdots,|u_n|)^{\mathrm{T}}$,$\boldsymbol{u}$ 的范数为 $\|\boldsymbol{u}\|=\sqrt{\sum_{i=1}^{n}|u_i|^2}$.

对于四元数矩阵 $\boldsymbol{A}\in H^{n\times n}$ 和 $\boldsymbol{B}\in H^{n\times n}$,有

①$(\boldsymbol{AB})^*=\boldsymbol{B}^*\boldsymbol{A}^*$;

②若 $\boldsymbol{AB}=\boldsymbol{BA}=\boldsymbol{I}$,则称矩阵 $\boldsymbol{A}$ 是可逆的;

③若 $\boldsymbol{A}^*=\boldsymbol{A}$,则称矩阵 $\boldsymbol{A}$ 是 Hermitian 矩阵;

④对于 Hermitian 矩阵 $\boldsymbol{A}\in H^{n\times n}$,如果对于所有非零向量 $\boldsymbol{x}\in H^n$ 有 $\boldsymbol{x}^*\boldsymbol{Ax}>0$,那么称矩阵 $\boldsymbol{A}$ 是正定的;

⑤对于矩阵 $\boldsymbol{A}=(a_{ij})_{n\times n}\in H^{n\times n}$,则 $\boldsymbol{A}$ 的范数为 $\|\boldsymbol{A}\|=\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}|a_{ij}|^2}$.

1.2　模型描述及基本引理

本文考虑如下具有离散时滞和分布时滞的 QVNNs 模型:

$$\dot{\boldsymbol{q}}(t)=-\boldsymbol{Dq}(t)+\boldsymbol{Cg}(\boldsymbol{q}(t))+\boldsymbol{Ag}(\boldsymbol{q}(t-\tau))+\boldsymbol{B}\int_{t-\tau}^{t}\boldsymbol{g}(\boldsymbol{q}(s))\mathrm{d}s+\boldsymbol{I},t\geqslant 0. \tag{1}$$

式中,神经元的状态向量 $\boldsymbol{q}(t)=(q_1(t),q_2(t),\cdots,q_n(t))^{\mathrm{T}}\in H^n$,$q_i(t)\in H$;$\boldsymbol{q}(t)$ 的激活函数 $\boldsymbol{f}(\boldsymbol{q}(t))=(f_1(q_1(t)),f_2(q_2(t)),\cdots,f_n(q_n(t)))^{\mathrm{T}}\in H^n$;$\tau\geqslant 0$ 表示传输时滞;$\boldsymbol{D}=\mathrm{diag}\{d_1,d_2,\cdots,d_n\}\in R_d^{n\times n}$ 表示自反馈连接权矩阵,其中 $d_i>0$;$\boldsymbol{A}=(a_{ij})_{n\times n}\in H^{n\times n}$,$\boldsymbol{B}=(b_{ij})_{n\times n}\in H^{n\times n}$,$\boldsymbol{C}=(c_{ij})_{n\times n}\in H^{n\times n}$ 均为连接权矩阵;外部输入向量 $\boldsymbol{I}=(I_1,I_2,\cdots,I_n)^{\mathrm{T}}\in H^n$.

模型(1)的初始条件为:$\boldsymbol{q}(s)=\boldsymbol{\phi}(s)$,$s\in[-\theta,0]$. $\boldsymbol{\phi}(s)$ 为 $[-\theta,0]$ 上的向量函数,有界且连续.

令模型(1)为驱动神经网络,则响应神经网络模型如下:

$$\dot{\boldsymbol{m}}(t)=-\boldsymbol{Dm}(t)+\boldsymbol{Cg}(\boldsymbol{m}(t))+\boldsymbol{Ag}(\boldsymbol{m}(t-\tau))+\boldsymbol{B}\int_{t-\tau}^{t}\boldsymbol{g}(\boldsymbol{m}(s))\mathrm{d}s+\boldsymbol{I}+\boldsymbol{u}(t),t\geqslant 0. \tag{2}$$

模型(2)的初始条件 $\boldsymbol{m}(s)=\boldsymbol{\psi}(s)$,$s\in[-\theta,0]$,$\boldsymbol{\psi}(s)$ 为 $[-\theta,0]$ 上的向量函数,有界且连续.

令 $\boldsymbol{e}(t)=\boldsymbol{q}(t)-\boldsymbol{m}(t)$,$\boldsymbol{f}(\boldsymbol{e}(t))=\boldsymbol{g}(\boldsymbol{q}(t))-\boldsymbol{g}(\boldsymbol{m}(t))$,控制器 $\boldsymbol{u}(t)=\boldsymbol{K}_1\boldsymbol{e}(t)+\boldsymbol{K}_2\boldsymbol{e}(t-\tau)$,其中 $\boldsymbol{K}_i\in R^{n\times n}$ 为对角矩阵. 从而误差神经网络模型为:

$$\dot{\boldsymbol{e}}(t)=(\boldsymbol{K}_1-\boldsymbol{D})\boldsymbol{e}(t)+\boldsymbol{K}_2\boldsymbol{e}(t-\tau)+\boldsymbol{C}\boldsymbol{f}(\boldsymbol{e}(t))+\boldsymbol{A}\boldsymbol{f}(\boldsymbol{e}(t-\tau))+\boldsymbol{B}\int_{t-\tau}^{t}\boldsymbol{f}(\boldsymbol{e}(s))\mathrm{d}s,t\geqslant 0, \tag{3}$$

初始条件为:

$$\boldsymbol{e}(s)=\boldsymbol{\varphi}(s)=\boldsymbol{\phi}(s)-\boldsymbol{\psi}(s),s\in[-\theta,0].$$

本文结论满足以下假设:

(H_1)激励函数$f_j(\cdot)$是连续的,并且对任意$\alpha_1,\alpha_2\in H$, $j=1,2,\cdots n$,

$$|f_j(\alpha_1)-f_j(\alpha_2)|\leqslant\eta_j|\alpha_1-\alpha_2|,$$

式中,η_j为常数,定义$\boldsymbol{\Gamma}=\mathrm{diag}(\eta_1,\eta_2,\cdots\eta_n)$.

定义 1[15]　如果误差四元数神经网络(3)是全局稳定的,那么就说驱动四元数神经网络(1)和响应四元数神经网络(2)是全局同步的.

定义 2[13]　对于模型(3)的任意解$\boldsymbol{e}(t)=(e_1(t),e_2(t),\cdots e_n(t))$,如果存在常数$M>0$,使得

$$\|\boldsymbol{e}(t)\|\leqslant M\sup_{s\in[-\theta,0]}\|\boldsymbol{\varphi}(s)-\boldsymbol{\psi}(s)\|$$

成立,那么就说四元数神经网络(3)是全局稳定的.

引理 1[13]　对任意常数矩阵$\boldsymbol{W}\in H^{n\times n}$,$\boldsymbol{W}>0$和向量函数$\boldsymbol{\omega}:[a,b]\to H^n$,$a<b$有

$$\left(\int_a^b\boldsymbol{\omega}(s)\mathrm{d}s\right)^*\boldsymbol{W}\left(\int_a^b\boldsymbol{\omega}(s)\mathrm{d}s\right)\leqslant(b-a)\int_a^b\boldsymbol{\omega}^*(s)\boldsymbol{W}\boldsymbol{\omega}(s)\mathrm{d}s.$$

2　主要结果

定理　在H_1成立的条件下,若存在正定Hermitian矩阵$\boldsymbol{P}_1,\boldsymbol{P}_2,\boldsymbol{P}_3,\boldsymbol{R}\in H^{n\times n}$,正定对角矩阵$\boldsymbol{Q}\in R^{n\times n}$和矩阵$\boldsymbol{N}_1,\boldsymbol{N}_2,\boldsymbol{N}_3\in H^{n\times n}$,使得下式成立

$$\begin{bmatrix}\boldsymbol{\Omega}_{11} & \boldsymbol{\Omega}_{12} & \boldsymbol{\Omega}_{13} & \boldsymbol{\Omega}_{14} & \boldsymbol{\Omega}_{15}\\ * & \boldsymbol{\Omega}_{22} & \boldsymbol{\Omega}_{23} & \boldsymbol{N}_1^*\boldsymbol{C} & \boldsymbol{N}_1^*\boldsymbol{A}\\ * & * & \boldsymbol{\Omega}_{33} & \boldsymbol{N}_3^*\boldsymbol{C} & \boldsymbol{N}_3^*\boldsymbol{A}\\ * & * & * & \tau^2\boldsymbol{P}_3+\boldsymbol{P}_2-\boldsymbol{Q} & 0\\ * & * & * & * & -\boldsymbol{P}_2\\ * & * & * & * & *\end{bmatrix}<0. \tag{4}$$

式中

$$\boldsymbol{\Omega}_{11}=\boldsymbol{\Gamma}^*\boldsymbol{Q}\boldsymbol{\Gamma}+(\boldsymbol{K}_1-\boldsymbol{D})\boldsymbol{P}_1+\boldsymbol{P}_1(\boldsymbol{K}_1-\boldsymbol{D})+\boldsymbol{R}+(\boldsymbol{K}_1-\boldsymbol{D})\boldsymbol{N}_2+\boldsymbol{N}_2^*(\boldsymbol{K}_1-\boldsymbol{D}),$$

$$\boldsymbol{\Omega}_{12}=(\boldsymbol{K}_1-\boldsymbol{D})\boldsymbol{N}_1-\boldsymbol{N}_2^*,\boldsymbol{\Omega}_{13}=\boldsymbol{P}_1\boldsymbol{K}_2+(\boldsymbol{K}_1-\boldsymbol{D})\boldsymbol{N}_3+\boldsymbol{N}_2^*\boldsymbol{K}_2,$$

$$\boldsymbol{\Omega}_{14}=\boldsymbol{P}_1\boldsymbol{C}+\boldsymbol{N}_2^*\boldsymbol{C},\boldsymbol{\Omega}_{15}=\boldsymbol{P}_1\boldsymbol{A}+\boldsymbol{N}_2^*\boldsymbol{A},\boldsymbol{\Omega}_{16}=\boldsymbol{P}_1\boldsymbol{B}+\boldsymbol{N}_2^*\boldsymbol{B},$$

$$\boldsymbol{\Omega}_{22}=-\boldsymbol{N}_1-\boldsymbol{N}_1^*,\boldsymbol{\Omega}_{23}=-\boldsymbol{N}_3+\boldsymbol{N}_1^*\boldsymbol{K}_2,$$

$$\boldsymbol{\Omega}_{33}=-\boldsymbol{R}-\boldsymbol{K}_2\boldsymbol{N}_3-\boldsymbol{N}_3^*\boldsymbol{K}_2,$$

则四元数神经网络模型(1)和四元数神经网络模型(2)是全局同步的.

注:本文中“*”是表示矩阵的共轭转置.

证明　构造Lyapunov-Krasovskii泛函:

$$V(t)=V_1(t)+V_2(t)+V_3(t)+V_4(t),$$

式中

$$V_1(t)=\boldsymbol{e}^*(t)\boldsymbol{P}_1\boldsymbol{e}(t),$$

$$V_2(t)=\int_{t-\tau}^{t}\boldsymbol{e}^*(s)\boldsymbol{R}\boldsymbol{e}(s)\mathrm{d}s,$$

$$V_3(t)=\int_{t-\tau}^{t}\boldsymbol{f}^*(\boldsymbol{e}(s))\boldsymbol{P}_2\boldsymbol{f}(\boldsymbol{e}(s))\mathrm{d}s,$$

$$V_4(t)=\tau\int_{-\tau}^{0}\int_{t+s}^{t}\boldsymbol{f}^*(\boldsymbol{e}(\theta))\boldsymbol{P}_3\boldsymbol{f}(\boldsymbol{e}(\theta))\mathrm{d}\theta\mathrm{d}s.$$

沿着误差神经网络模型(3)求导,应用引理 1 得出

$$\boldsymbol{D}^+V_1(t)=\dot{\boldsymbol{e}}^*(t)\boldsymbol{P}_1\boldsymbol{e}(t)+\boldsymbol{e}^*(t)\boldsymbol{P}_1\dot{\boldsymbol{e}}(t)=$$
$$\left[(\boldsymbol{K}_1-\boldsymbol{D})\boldsymbol{e}(t)+\boldsymbol{K}_2\boldsymbol{e}(t-\tau)+\boldsymbol{C}\boldsymbol{f}(\boldsymbol{e}(t))+\boldsymbol{A}\boldsymbol{f}(\boldsymbol{e}(t-\tau))+\boldsymbol{B}\int_{t-\tau}^{t}\boldsymbol{f}(\boldsymbol{e}(s))\mathrm{d}s\right]^*\boldsymbol{P}_1\boldsymbol{e}(t)+$$
$$\boldsymbol{e}^*(t)\boldsymbol{P}_1\left[(\boldsymbol{K}_1-\boldsymbol{D})\boldsymbol{e}(t)+\boldsymbol{K}_2\boldsymbol{e}(t-\tau)+\boldsymbol{C}\boldsymbol{f}(\boldsymbol{e}(t))+\boldsymbol{A}\boldsymbol{f}(\boldsymbol{e}(t-\tau))+\boldsymbol{B}\int_{t-\tau}^{t}\boldsymbol{f}(\boldsymbol{e}(s))\mathrm{d}s\right]^*. \tag{5}$$

$$\boldsymbol{D}^+V_2(t)=\boldsymbol{e}^*(t)\boldsymbol{R}\boldsymbol{e}(t)-\boldsymbol{e}^*(t-\tau)\boldsymbol{R}\boldsymbol{e}(t-\tau). \tag{6}$$

$$\boldsymbol{D}^+V_3(t)=\boldsymbol{f}^*(\boldsymbol{e}(t))\boldsymbol{P}_2\boldsymbol{f}(\boldsymbol{e}(t))-\boldsymbol{f}^*(\boldsymbol{e}(t-\tau))\boldsymbol{P}_2\boldsymbol{f}(\boldsymbol{e}(t-\tau)). \tag{7}$$

$$\boldsymbol{D}^+V_4(t)=\tau^2\boldsymbol{f}^*(\boldsymbol{e}(t))\boldsymbol{P}_3\boldsymbol{f}(\boldsymbol{e}(t))-\tau\int_{-\tau}^{0}\boldsymbol{f}^*(\boldsymbol{e}(t+s))\boldsymbol{P}_3\boldsymbol{f}(\boldsymbol{e}(t+s))\mathrm{d}s=$$
$$\tau^2\boldsymbol{f}^*(\boldsymbol{e}(t))\boldsymbol{P}_3\boldsymbol{f}(\boldsymbol{e}(t))-\tau\int_{t-\tau}^{t}\boldsymbol{f}^*(\boldsymbol{e}(s))\boldsymbol{P}_3\boldsymbol{f}(\boldsymbol{e}(s))\mathrm{d}s\leqslant$$
$$\tau^2\boldsymbol{f}^*(\boldsymbol{e}(t))\boldsymbol{P}_3\boldsymbol{f}(\boldsymbol{e}(t))-\int_{t-\tau}^{t}\boldsymbol{f}^*(\boldsymbol{e}(s))\boldsymbol{P}_3\boldsymbol{f}(\boldsymbol{e}(s))\mathrm{d}s. \tag{8}$$

将式(5)、(6)、(7)和(8)相加可得

$$\boldsymbol{D}^+V(t)\leqslant\left[(\boldsymbol{K}_1-\boldsymbol{D})\boldsymbol{e}(t)+\boldsymbol{K}_2\boldsymbol{e}(t-\tau)+\boldsymbol{C}\boldsymbol{f}(\boldsymbol{e}(t))+\boldsymbol{A}\boldsymbol{f}(\boldsymbol{e}(t-\tau))+\boldsymbol{B}\int_{t-\tau}^{t}\boldsymbol{f}(\boldsymbol{e}(s))\mathrm{d}s\right]^*\boldsymbol{P}_1\boldsymbol{e}(t)+$$
$$\boldsymbol{e}^*(t)\boldsymbol{P}_1\left[(\boldsymbol{K}_1-\boldsymbol{D})\boldsymbol{e}(t)+\boldsymbol{K}_2\boldsymbol{e}(t-\tau)+\boldsymbol{C}\boldsymbol{f}(\boldsymbol{e}(t))+\boldsymbol{A}\boldsymbol{f}(\boldsymbol{e}(t-\tau))+\boldsymbol{B}\int_{t-\tau}^{t}\boldsymbol{f}(\boldsymbol{e}(s))\mathrm{d}s\right]+$$
$$\boldsymbol{e}^*(t)\boldsymbol{R}\boldsymbol{e}(t)-\boldsymbol{e}^*(t-\tau)\boldsymbol{R}\boldsymbol{e}(t-\tau)+\boldsymbol{f}^*(\boldsymbol{e}(t))\boldsymbol{P}_2\boldsymbol{f}(\boldsymbol{e}(t))+\boldsymbol{f}^*(\boldsymbol{e}(t-\tau))\boldsymbol{P}_2\boldsymbol{f}(\boldsymbol{e}(t-\tau))+$$
$$\tau^2\boldsymbol{f}^*(\boldsymbol{e}(t))\boldsymbol{P}_3\boldsymbol{f}(\boldsymbol{e}(t))-\int_{t-\tau}^{t}\boldsymbol{f}^*(\boldsymbol{e}(s))\mathrm{d}s\boldsymbol{P}_3\int_{t-\tau}^{t}\boldsymbol{f}(\boldsymbol{e}(s))\mathrm{d}s=$$
$$\boldsymbol{e}^*(t)[(\boldsymbol{K}_1-\boldsymbol{D})\boldsymbol{P}_1+\boldsymbol{P}_1(\boldsymbol{K}_1-\boldsymbol{D})+\boldsymbol{R}]\boldsymbol{e}(t)+\boldsymbol{e}^*(t)\boldsymbol{P}_1\boldsymbol{K}_2\boldsymbol{e}(t-\tau)+\boldsymbol{e}^*(t-\tau)\boldsymbol{K}_2\boldsymbol{P}_1\boldsymbol{e}(t)-$$
$$\boldsymbol{e}^*(t-\tau)\boldsymbol{R}\boldsymbol{e}(t-\tau)+\boldsymbol{e}^*(t)\boldsymbol{P}_1\boldsymbol{C}\boldsymbol{f}(\boldsymbol{e}(t))+$$
$$\boldsymbol{f}^*(\boldsymbol{e}(t))\boldsymbol{C}^*\boldsymbol{P}_1\boldsymbol{e}(t)+\boldsymbol{f}^*(\boldsymbol{e}(t-\tau))\boldsymbol{A}^*\boldsymbol{P}_1\boldsymbol{e}(t)+\boldsymbol{e}^*(t)\boldsymbol{P}_1\boldsymbol{A}\boldsymbol{f}(\boldsymbol{e}(t-\tau))+$$
$$\int_{t-\tau}^{t}\boldsymbol{f}^*(\boldsymbol{e}(t-\tau))\mathrm{d}s\boldsymbol{B}^*\boldsymbol{P}_1\boldsymbol{e}(t)+\boldsymbol{e}^*(t)\boldsymbol{P}_1\boldsymbol{B}\int_{t-\tau}^{t}\boldsymbol{f}(\boldsymbol{e}(s))\mathrm{d}s+\boldsymbol{f}^*(\boldsymbol{e}(t))(\tau^2\boldsymbol{P}_3+\boldsymbol{P}_2)\boldsymbol{f}(\boldsymbol{e}(t))-$$
$$\boldsymbol{f}^*(\boldsymbol{e}(t-\tau))\boldsymbol{P}_2\boldsymbol{f}(\boldsymbol{e}(t-\tau))-\int_{t-\tau}^{t}\boldsymbol{f}^*(\boldsymbol{e}(s))\mathrm{d}s\boldsymbol{P}_3\int_{t-\tau}^{t}\boldsymbol{f}(\boldsymbol{e}(s))\mathrm{d}s. \tag{9}$$

由假设(H_1)有

$$\boldsymbol{f}^*(\boldsymbol{e}(t))\boldsymbol{Q}\boldsymbol{f}(\boldsymbol{e}(t))-\boldsymbol{e}^*(t)\boldsymbol{\Gamma}^*\boldsymbol{Q}\boldsymbol{\Gamma}\boldsymbol{e}(t)\leqslant 0. \tag{10}$$

且由式(3)得到

$$0=\left[-\dot{\boldsymbol{e}}(t)+(\boldsymbol{K}_1-\boldsymbol{D})\boldsymbol{e}(t)+\boldsymbol{K}_2\boldsymbol{e}(t-\tau)+\boldsymbol{C}\boldsymbol{f}(\boldsymbol{e}(t))+\boldsymbol{A}\boldsymbol{f}(\boldsymbol{e}(t-\tau))+\boldsymbol{B}\int_{t-\tau}^{t}\boldsymbol{f}(\boldsymbol{e}(s))\mathrm{d}s\right]^*[\boldsymbol{N}_1\boldsymbol{e}(t)+$$
$$\boldsymbol{N}_2\dot{\boldsymbol{e}}(t)+\boldsymbol{N}_3\boldsymbol{e}(t-\tau)]+[\boldsymbol{N}_1\boldsymbol{e}(t)+\boldsymbol{N}_2\dot{\boldsymbol{e}}(t)+\boldsymbol{N}_3\boldsymbol{e}(t-\tau)]^*-$$
$$\left[-\dot{\boldsymbol{e}}(t)+(\boldsymbol{K}_1-\boldsymbol{D})\boldsymbol{e}(t)+\boldsymbol{K}_2\boldsymbol{e}(t-\tau)+\boldsymbol{C}\boldsymbol{f}(\boldsymbol{e}(t))+\boldsymbol{A}\boldsymbol{f}(\boldsymbol{e}(t-\tau))+\boldsymbol{B}\int_{t-\tau}^{t}\boldsymbol{f}(\boldsymbol{e}(s))\mathrm{d}s\right]. \tag{11}$$

将不等式(9),(10)和等式(11)相加得

$$\boldsymbol{D}^+V(t)\leqslant\boldsymbol{\pi}^*(t)\boldsymbol{\Omega}\boldsymbol{\pi}(t), \tag{12}$$

式中

$$\boldsymbol{\pi}(t)=\left(\boldsymbol{e}(t),\dot{\boldsymbol{e}}(t),\boldsymbol{e}(t-\tau),\boldsymbol{f}(\boldsymbol{e}(t)),\boldsymbol{f}(\boldsymbol{e}(t-\tau)),\int_{t-\tau}^{\tau}\boldsymbol{f}(\boldsymbol{e}(s))\mathrm{d}s\right)^*,$$

$$\boldsymbol{\Omega}=\begin{bmatrix}\boldsymbol{\Omega}_{11} & \boldsymbol{\Omega}_{12} & \boldsymbol{\Omega}_{13} & \boldsymbol{\Omega}_{14} & \boldsymbol{\Omega}_{15} & \boldsymbol{\Omega}_{16}\\ * & \boldsymbol{\Omega}_{22} & \boldsymbol{\Omega}_{23} & \boldsymbol{N}_1^*\boldsymbol{C} & \boldsymbol{N}_1^*\boldsymbol{A} & \boldsymbol{N}_1^*\boldsymbol{B}\\ * & * & \boldsymbol{\Omega}_{33} & \boldsymbol{N}_3^*\boldsymbol{C} & \boldsymbol{N}_3^*\boldsymbol{A} & \boldsymbol{N}_3^*\boldsymbol{B}\\ * & * & * & \tau^2\boldsymbol{P}_3+\boldsymbol{P}_2-\boldsymbol{Q} & 0 & 0\\ * & * & * & * & -\boldsymbol{P}_2 & 0\\ * & * & * & * & * & -\boldsymbol{P}_3\end{bmatrix},$$

其中, $\boldsymbol{\Omega}_{11}=\boldsymbol{\Gamma}^{*}\boldsymbol{Q}\boldsymbol{\Gamma}+(\boldsymbol{K}_1-\boldsymbol{D})\boldsymbol{P}_1+\boldsymbol{P}_1(\boldsymbol{K}_1-\boldsymbol{D})+\boldsymbol{R}+(\boldsymbol{K}_1-\boldsymbol{D})\boldsymbol{N}_2+\boldsymbol{N}_2^{*}(\boldsymbol{K}_1-\boldsymbol{D})$,

$\boldsymbol{\Omega}_{12}=(\boldsymbol{K}_1-\boldsymbol{D})\boldsymbol{N}_1-\boldsymbol{N}_2^{*}$, $\boldsymbol{\Omega}_{13}=\boldsymbol{P}_1\boldsymbol{K}_2+(\boldsymbol{K}_1-\boldsymbol{D})\boldsymbol{N}_3+\boldsymbol{N}_2^{*}\boldsymbol{K}_2$,

$\boldsymbol{\Omega}_{14}=\boldsymbol{P}_1\boldsymbol{C}+\boldsymbol{N}_2^{*}\boldsymbol{C}$, $\boldsymbol{\Omega}_{15}=\boldsymbol{P}_1\boldsymbol{A}+\boldsymbol{N}_2^{*}\boldsymbol{A}$, $\boldsymbol{\Omega}_{16}=\boldsymbol{P}_1\boldsymbol{B}+\boldsymbol{N}_2^{*}\boldsymbol{B}$,

$\boldsymbol{\Omega}_{22}=-\boldsymbol{N}_1-\boldsymbol{N}_1^{*}$, $\boldsymbol{\Omega}_{23}=-\boldsymbol{N}_3+\boldsymbol{N}_1^{*}\boldsymbol{K}_2$,

$\boldsymbol{\Omega}_{33}=-\boldsymbol{R}-\boldsymbol{K}_2\boldsymbol{N}_3-\boldsymbol{N}_3^{*}\boldsymbol{K}_2$.

故根据式(4)和(12)得

$$\boldsymbol{D}^{+}V(t)\leqslant 0,\ t\geqslant 0. \tag{13}$$

故 $V(t)$ 在 $t\geqslant 0$ 上为单调非增函数,于是有

$$\begin{aligned}V(t)\leqslant V(0)=\boldsymbol{e}^{*}(0)\boldsymbol{P}_1\boldsymbol{e}(0)+\int_{-\tau}^{0}\boldsymbol{e}^{*}(s)\boldsymbol{R}\boldsymbol{e}(s)\mathrm{d}s+\int_{-\tau}^{0}\boldsymbol{f}^{*}(\boldsymbol{e}(s))\boldsymbol{P}_2\boldsymbol{f}(\boldsymbol{e}(s))\mathrm{d}s+\\ \tau\int_{-\tau}^{0}\int_{s}^{0}\boldsymbol{f}^{*}(\boldsymbol{e}(\theta))\boldsymbol{P}_3\boldsymbol{f}(\boldsymbol{e}(\theta))\mathrm{d}\theta\mathrm{d}s\leqslant\\ (\|\boldsymbol{P}_1\|+\tau\|\boldsymbol{R}\|+\tau\|\boldsymbol{P}_2\|+\tau^3\|\boldsymbol{P}_3\|)(\sup_{s\in[-\theta,0]}\|\boldsymbol{\varphi}(s)\|)^2=M_1(\sup_{s\in[-\theta,0]}\|\boldsymbol{\varphi}(s)\|)^2,\end{aligned} \tag{14}$$

式中

$$M_1=\|\boldsymbol{P}_1\|+\tau\|\boldsymbol{R}\|+\tau\|\boldsymbol{P}_2\|+\tau^3\|\boldsymbol{P}_3\|.$$

由于

$$V(t)\geqslant V_1(t)\geqslant\lambda_{\min}(\boldsymbol{P}_1)\|\boldsymbol{e}(t)\|^2, \tag{15}$$

则

$$\|\boldsymbol{e}(t)\|\leqslant\sqrt{\frac{V(t)}{\lambda_{\min}(\boldsymbol{P}_1)}}=\sqrt{\frac{M_1}{\lambda_{\min}(\boldsymbol{P}_1)}}\sup_{s\in[-\theta,0]}\|\boldsymbol{\varphi}(s)\|,\ t\geqslant 0, \tag{16}$$

即

$$\|\boldsymbol{e}(t)\|\leqslant M\sup_{s\in[-\theta,0]}\|\boldsymbol{\varphi}(s)\|,\ t\geqslant 0, \tag{17}$$

式中

$$M=\frac{\sqrt{M_1}}{\sqrt{\lambda_{\min}(\boldsymbol{P}_1)}}.$$

由定义 1 和定义 2,我们知道模型(3)是全局稳定的,所以模型(1)和模型(2)是全局同步的. 至此,定理证毕.

如果在不考虑分布时滞对系统的影响条件下,则模型(1)可表示为

$$\dot{\boldsymbol{q}}(t)=-\boldsymbol{D}\boldsymbol{q}(t)+\boldsymbol{C}g(\boldsymbol{q}(t))+\boldsymbol{A}g(\boldsymbol{q}(t-\tau))+\boldsymbol{I}(t),\ t\geqslant 0. \tag{18}$$

响应模型表示为:

$$\dot{\boldsymbol{m}}(t)=-\boldsymbol{D}\boldsymbol{m}(t)+\boldsymbol{C}\boldsymbol{g}(\boldsymbol{m}(t))+\boldsymbol{A}\boldsymbol{g}(\boldsymbol{m}(t-\tau))+\boldsymbol{I}(t)+\boldsymbol{u}(t),\ t\geqslant 0, \tag{19}$$

式中,控制器 $\boldsymbol{u}(t)=-\boldsymbol{K}_1\boldsymbol{e}(t)$.于是由驱动神经网络(18)和响应神经网络(19)可得误差神经网络:

$$\dot{\boldsymbol{e}}(t)=(\boldsymbol{K}_1-\boldsymbol{D})\boldsymbol{e}(t)+\boldsymbol{C}\boldsymbol{f}(\boldsymbol{e}(t))+\boldsymbol{A}\,\boldsymbol{f}(\boldsymbol{e}(t-\tau)),\ t\geqslant 0. \tag{20}$$

相应初始条件与前面模型一致,由前面定理,可以得到如下推论.

推论　在(H_1)成立的条件下,若存在正定 Hermitian 矩阵 $\boldsymbol{P}_1,\boldsymbol{P}_2,\boldsymbol{P}_3,\boldsymbol{R}\in H^{n\times n}$,正定对角矩阵 $\boldsymbol{Q}\in R^{n\times n}$ 和矩阵 $\boldsymbol{N}_1,\boldsymbol{N}_2\in H^{n\times n}$,使得不等式

$$\begin{bmatrix}\boldsymbol{\xi}_{11} & \boldsymbol{\xi}_{12} & \boldsymbol{\xi}_{13} & \boldsymbol{\xi}_{14} & \boldsymbol{\xi}_{15}\\ * & \boldsymbol{\xi}_{22} & \boldsymbol{N}_2^{*}\boldsymbol{C} & \boldsymbol{N}_2^{*}\boldsymbol{A} & 0\\ * & * & \boldsymbol{\xi}_{33} & 0 & 0\\ * & * & * & -\boldsymbol{P}_2 & 0\\ * & * & * & * & -\boldsymbol{P}_3\end{bmatrix}<0 \tag{21}$$

成立,其中

$$\boldsymbol{\xi}_{11}=\boldsymbol{\Gamma}^{*}\boldsymbol{Q}\boldsymbol{\Gamma}+(\boldsymbol{K}_1-\boldsymbol{D})\boldsymbol{P}_1+\boldsymbol{P}_1(\boldsymbol{K}_1-\boldsymbol{D})+\boldsymbol{R},$$
$$\boldsymbol{\xi}_{12}=-\boldsymbol{N}_1^{*}+(\boldsymbol{K}_1-\boldsymbol{D})\boldsymbol{N}_2,\boldsymbol{\xi}_{13}=\boldsymbol{P}_1\boldsymbol{C}+\boldsymbol{N}_1^{*}\boldsymbol{C},$$
$$\boldsymbol{\xi}_{14}=\boldsymbol{P}_1\boldsymbol{A}+\boldsymbol{N}_1^{*}\boldsymbol{A},\boldsymbol{\xi}_{22}=-\boldsymbol{N}_2-\boldsymbol{N}_2^{*},$$
$$\boldsymbol{\xi}_{33}=\tau^2\boldsymbol{P}_3+\boldsymbol{P}_2-\boldsymbol{Q}.$$

则四元数神经网络(18)和(19)是全局同步的. 对比文献[13]和[14],本文在神经网络模型中引入离散时滞和分布时滞并且将复值神经网络的研究方法推广到四元数神经网络,改进了以上文献的工作.

3 结论

本文研究了一类具有离散时滞和分布时滞的 QVNNs 的同步性问题. 通过构造合适的李雅普诺夫函数,运用了不等式技巧和自由权矩阵方法,得到了误差模型的全局稳定性,从而就能得到驱动模型和响应模型是全局同步的,并得到了判定四元数神经网络全局同步性的不等式判断依据和相关推论.

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